MODELLING A SAG GRINDING SYSTEM THROUGH MULTIPLES REGRESSIONS

Mauricio VILLANUEVA[1], Christian CALDERÓN[1], Manuel SALDAÑA[2,3], Norman TORO[3]

[1] *Departamento de Ingeniería Industrial, Facultad de Ingeniería y Ciencias Geológicas, Universidad Católica del Norte, Antofagasta 1270709, Chile, mvillanueva@mineracentinela.cl, ccaldel12@gmail.com*

[2] *Faculty of Engineering and Architecture, Universidad Arturo Prat, Antofagasta 1244260, Chile, masaldana@unap.cl*

[3] *Departamento de Ingeniería Metalúrgica y Minas, Facultad de Ingeniería y Ciencias Geológicas, Universidad Católica del Norte, Antofagasta 1270709, Chile, ntoro@ucn.cl*

https://doi.org/10.37904/metal.2020.3665

Abstract

Due to the constant growth of the copper industry, the increase in production costs and complexity in the composition of the feed of the production processes that make up the industry, the analysis of alternatives that improve efficiency by studying the dynamics of the processes represents a significant cost reduction. Then, the generation of analytical models that represent the dynamic behaviour of production processes has the potential to contribute to generating a better understanding of the operating parameters that have a greater impact on the response (s), in addition to identifying operating restrictions and optimal levels of operation. The present work developed a digital model of the SAG milling process by generating multiple regression and quadratic regression models. The relationships between 22 operational variables with production in tons per hour were sampled, and after analysing the impact of the independent variables on the response, water feeding, sump level, percentage of solids in feeding, pebbles and hardness were maintained for fit the analytical models. The multiple linear regression model presents a good fit to the operational data (85.4%), however, the inclusion of the interactions and the quadratic effects of the variables increases the coefficient of determination (93.2%).

Keywords: SAG milling, modelling, mineral processing, mathematical models

1. INTRODUCTION

There is constant growth in the copper industry and production has been increasing in recent years [1], increasing from 20 million tons in 2017 to 21 (rounded) in 2018, while a more recent report generated by the International Copper Study Group [2] indicates that since 1900, the production of copper minerals worldwide has grown an average of 3.2% per year, reaching 20.6 million tons in 2018, with an increase in the production of concentrates of 31.5% and solvent-electrodeposition extraction in 19.5% [3]. Chile is the main producer of copper worldwide with a 29% participation and with 23% of the reserves of this commodity [4]. Within the national territory there are 3,817 copper mineral deposits [5], where their exploitation represents 91.1 % of the composition of exports by the mining market in 2019 [4].

Copper oxides that are processed by hydrometallurgy are increasingly scarce in Chile (Copper oxides will decrease from 30.8% in 2015 to 12% in 2027), while copper sulphides is more abundant [6]. On fact, 39.2% of fine copper production occurs through the hydrometallurgical processes, while the majority of production (60.8%) is by flotation processes. A report by Chilean Copper Commission, COCHILCO (by its acronym in Spanish) [6] proposes a constant increase in the production of copper concentrates in Chile, where it's indicated that from 2014 to 2026 it will almost double, being 88% of the national mining production, which means an increase from 3.9 to 5.4 million tons of concentrate. However, flotation processes generate large environmental liabilities, such as tailings dams [1]. It is estimated that, in the country, for each ton of Cu obtained by flotation processes, 151 tons of tailings are generated [6]. Currently, there are 92 mining operations defined as mining environmental liabilities, a decrease of these deposits year-to-year is expected [7].

On the planet most of the copper minerals correspond to sulphides and a minor part to oxides [8]. The mining industry has traditionally operated in two ways, pyrometallurgy if it is sulphided minerals, composed by the flotation, smelting and electro-refining processes. While in the hydrometallurgical processes, it has worked mainly with oxidized minerals, composed by the leaching, solvent extraction and electro-extraction processes [9]. Both working mechanisms have proven to be profitable in industry, however pyrometallurgy has the main disadvantage of making SO_2 emissions into the atmosphere, generating serious environmental problems [10]. As part of the sulphide mineral processing, the comminution process is a key stage, since it is where most of the energy invested for process the mineral is concentrated [11]. Then, the SAG milling process consists of reducing the size of the particles through the use of large rotating equipment or cylindrical mills, where water is added to the mineralized material in sufficient quantities to form a milky fluid, and the reagents necessary for carry out downstream processes.

Considering the above, the analysis of alternatives that improve the efficiency of production processes involves significant reduction in costs, considering the difficult situation facing the industry, where contractions are occurring worldwide as a result of the contingency [12]. Operational planning that considers the leaching of both oxidized and sulphide (secondary) minerals could improve efficiency in the use of resources, decreasing the average production costs and increase the productivity.

2. MATERIALS AND METHODS

Regression analyzes are part of statistics that investigate the relationship between two or more variables related in a non-deterministic way [13]. Simple linear regression analyzes relate a single independent variable to a response variable, while multiple linear regression analyzes allow the generation of a linear model in which the value of the dependent variable or response (Y) is determined from a set of independent variables called regressors (X_1,X_2,X_3...). Then, multiple linear regression models are an extension of simple linear regression and multiple non-linear regression models incorporate the interaction and curvature of independent variables. Multiple regression models can be used to predict the value of the dependent variable, to evaluate the influence that predictors have on it (the latter must be analyzed with caution in order not to misinterpret cause-effect) or to optimize the response bounded to the sampling domain [15–17].

Multiple regression models can be expressed as presented in equation (1).

$$Y = \left(\beta_0 + \sum_{i=1}^{n} \beta_i X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} X_i X_j\right) + e_i \tag{1}$$

where:

β_0 - ordinate at the origin

Y - dependent variable

X - independent variables

e_i - residual or error, the difference between the observed value and estimated by the model

The database used in this research comes from a plant that processes copper sulphide minerals, collected with hourly frequency. The period March-August 2019 was sampled, and 23 variables were measured, including the response studied (production in tons per hour, TpH).

3. DEVELOPMENT OF ANALYTICAL MODELS

The use of mathematical models as instruments for evaluating alternatives and modelling of complex processes is becoming increasingly important in the field of engineering, with an increasingly relevant role as an aid in decision-making in the management of mineral processing. A total of 23 operational variables and parameters were sampled, those that include the speed and pressure of the mill, power, water feeding, soluble copper grade, carbonate grade, percentage of solids in the feed, mineral hardness, sump level, granulometry, pebbles, liner age, among others process parameters such as critical noise or control variables of the feeders. Then, filtering variables and/or parameters that don't have a direct relationship with the response, the sample size is deficient or tends to remain constant, the principal explanatory variables chosen for mathematical modelling are shown in **Figure 1** (water in feeding, sump level, hardness, solids in feeding and pebbles), while the distributions of these explanatory variables is presented in **Figure 2**.

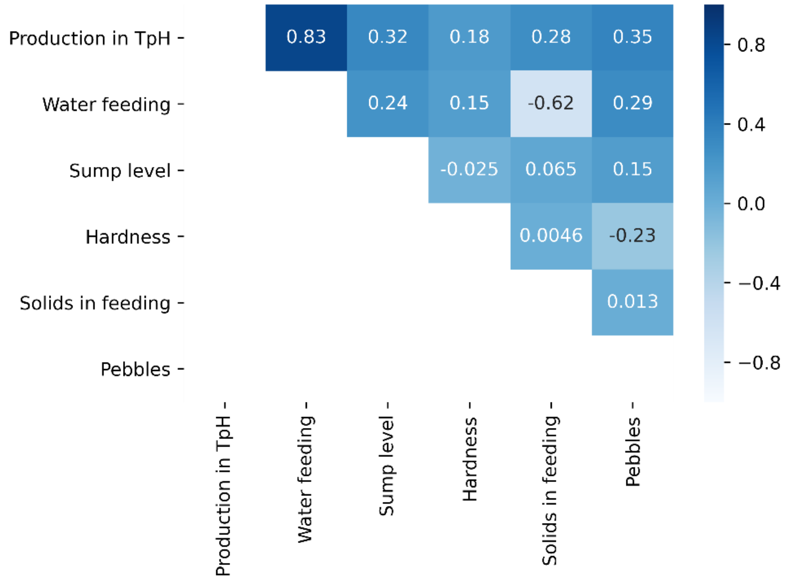


Figure 1 Correlation plot of the operational variables of the SAG milling process

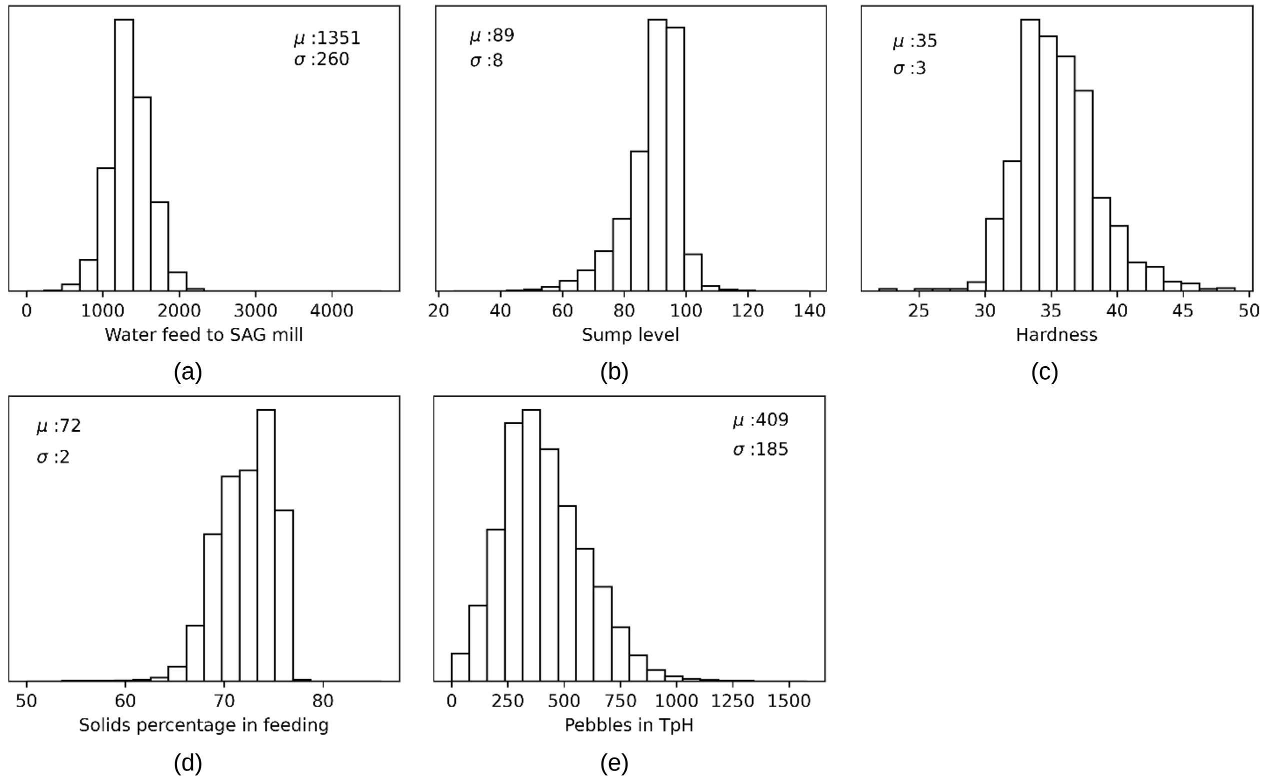


Figure 1 Distributions of the operational variables water feeding (a), sump level (b), mineral hardness in the feeding (c), solid percentage in feeding (d) and pebbles in TpH (e)

The correlation plot shown in **Figure 1** indicate a high linear correlation between water feeding and the response (Production in TpH), while that don't exist a linear correlation between the hardness and solids in feeding variables with production. The distribution of explanatory variables (see **Figure 2**) indicates that water feed to SAG mill has a normal distribution with a mean of 1351 m^3/h approximately, sump level has a negative skew distribution with a mean of 89 m^3/h, hardness has a positive skew distribution with a mean of 35, solids percentage in feeding has a negative skew distribution with a mean of 72% and finally, pebbles has a positive skew distribution with a mean of 409 TpH.

After the analysis of correlations shown in **Figure 1**, multiple regression models and quadratic regression models are generated. Then, the fitting of multiple regression model to represent the dynamics of the SAG milling process presents good indicators of goodness of fit. The model presents a good fit (R^2 = 85.4%) and all the variables considered are significant (p <0.05, see **Table 1**) under the set of sampled values, while the F statistic is 2.429×10^6.

Table 1 Results of the multiple linear regression model

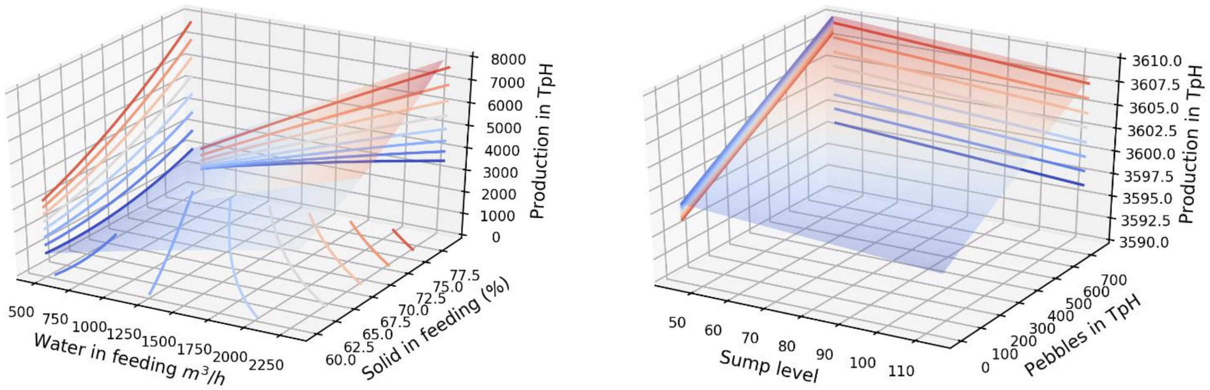
Factor	coefficient	t	p-value
Intercept	-1.12E+04	-2790.923	0.000
Water in feeding	2.3977	4124.138	0.000
Sump level	0.2694	19.334	0.000
Hardness	3.0049	74.947	0.000
Solids in feeding	156.4508	3177.553	0.000
Pebbles	0.0593	86.812	0.000

In addition to the adjustment of the multiple linear regression model presented in **Table 1**, a quadratic regression model is developed, incorporating the effects of interaction and curvature of the variables considered in the model presented in **Table 2**. The quadratic model indicates that the interactions water in feeding with sump level and solid in feeding are significant, in addition to the interaction solids in feeding with pebbles, contribute to explain the response variable. On the other hand, the curvature of the variables water and solids in feeding also contribute to explain the response. The quadratic model presents a better fit than the model linear (R^2 = 93.2%) and the statistic p-value (<0.05, see **Table 2**), both of each of the parameters that make up the model, and the aggregate model, validates it. The F statistic is 5.745×10^6.

Table 2 Results of the adjustment of the quadratic regression model

Factor	coefficient	t	p-value
Intercept	21790	605.781	0.000
Water in feeding	-4.5608	-426.139	0.000
Sump level	-0.1481	-3.512	0.000
Hardness	1.3239	51.931	0.000
Solids in feeding	-639.2631	-693.913	0.000
Pebbles	-0.4342	-44.098	0.000
Water in feeding × Sump level	0.0001	4.349	0.000
Water in feeding × Solids in feeding	0.103	800.718	0.000
Solids in feeding × Pebble	0.006	43.724	0.000
Water in feeding^2	-0.0001	-113.391	0.000
Solids in feeding^2	4.632	759.521	0.000

Finally, the response surface designs for the quadratic model based on the independent variables indicate that production increases at high levels of solids and water in feeding (see **Figure 3.a**), at high feed of pebbles in TpH (see **Figure 3.b**), and low sump levels (see **Figure 3.b**).



(a) (b)

Figure 2 Response surface plots for production versus water and solids in feeding (a), and sump and pebbles level in TpH (b)

4. CONCLUSIONS

The mineral deposits are usually heterogeneous, which forces the production phase to evolve over time. In this research, the modelling of the dynamics of the SAG grinding system was considered, for which the impact of 22 operating variables on production in tons per hour was evaluated. The correlation between the variables was studied, filtering the variables that have a greater impact on the response, which are pressure, mill speed, percentage of solids in the feed, mineral hardness, and sump level. Multiple linear regression model and a quadratic model were generated, which represented significant adjustments to the sampled domain, with R^2 values of 85.4% and 93.2%, respectively. Multiple regression models prove to be a powerful tool in modelling the studied system, in addition to presenting the potential of using optimization algorithms to calculate the values that maximize the response.

Finally, the dynamics of the SAG milling process could be modelled, simulated, and optimized using conventional statistical models [18], machine learning techniques [19], Bayesian networks [20] or discrete event simulation framework [21].

REFERENCES

[1] FLANAGAN, D. M. Copper: in Mineral Commodity Summaries 2019, Virginia, USA: *U.S. Geological Survey*, 2019, pp. 52–53.

[2] INTERNATIONAL COPPER STUDY GROUP, The World Copper Factbook 2019, Lisbon, Portugal, 2019. [online]. [viewed 2019-09-23]. Available from: https://www.icsg.org/index.php/component/jdownloads/finish/170/2965

[3] INTERNATIONAL COPPER STUDY GROUP, Release of ICSG 2018 Statistical Yearbook, Lisbon, Portugal, 2019. [online]. [viewed 2019-12-21]. Available from: http://www.icsg.org/index.php/press-releases/finish/170-publications-press-releases/2983-2019-12-09-press-release-yearbook-2019

[4] CONSEJO MINERO, Cifras actualizadas de la minería, Santiago, Chile, 2019. [online]. [viewed 2020-03-15]. Available from: https://consejominero.cl/chile-pais-minero/cifras-actualizadas-de-la-mineria/

[5] SERVICIO NACIONAL DE GEOLOGÍA Y MINERÍA (SERNAGEOMIN), Anuario de la mineria de Chile 2017, Santiago, Chile, 2017. [online]. [viewed 2019-12-21]. Available from: https://www.sernageomin.cl/wp-content/uploads/2018/06/Anuario_2017.pdf

[6] COMISIÓN CHILENA DEL COBRE (COCHILCO), Sulfuros primarios: desafíos y oportunidades, Santiago, Chile, 2017. [online]. [viewed 2019-12-21]. Available from: https://www.cochilco.cl/Listado%20Temtico/sulfuros%20primarios_desaf%C3%ADos%20y%20oportunidades.pdf

[7] MORALES, C. La apuesta por la reutilización de escorias, *Revista Nueva Minería y Energía*, 2017. [Online]. [viewed 2019-10-29]. Available from: http://www.nuevamineria.com/revista/la-apuesta-por-la-reutilizacion-de-escorias/

[8] HERNÁNDEZ, P. C. Estudio del equilibrio sólido-líquido de sistemas acuosos de minerales de cobre con agua de mar, aplicado a procesos de lixiviación, *Doctoral Thesis*, Facultad de Ingeniería, Universidad de Antofagasta, Antofagasta, Chile, 2013. [Online]. [viewed 2019-10-29]. Available from: https://intranetua.uantof.cl/docs/carreras/tesis_pia_hernandez.pdf

[9] TORO, N., PÉREZ, K., SALDAÑA, M., JELDRES, R. I., JELDRES, M., CÁNOVAS, M. Dissolution of pure chalcopyrite with manganese nodules and waste water, *Journal of Materials Research and Technology*, 2020, vol. 9, no. 1, pp. 798–805.

[10] B. S. SOSA, R. B. BANDA-NORIEGA, E. M. GUERRERO, Industrias de fundición: Aspectos ambientales e indicadores de condición ambiental, *Revista de Metalurgia*, 2013, vol. 49, no. 1, pp. 5–19.

[11] COMISIÓN CHILENA DEL COBRE (COCHILCO), Caracterización de los costos de la gran minería del cobre, Santiago, Chile, 2015. [Online]. [viewed 2010-02-18]. Available from: https://www.cochilco.cl/Listado%20Temtico/Informe_caracterizacion_de_los_costos.pdf

[12] INTERNATIONAL COPPER STUDY GROUP, Copper: Preliminary Data for January 2020, Lisbon, Portugal, 2020. [Online]. [viewed 2010-02-18]. Available from: https://www.icsg.org/index.php/component/jdownloads/finish/114/3011?Itemid=

[13] DEVORE, J. Probability & Statistics for Engineering and the Sciences, Richard Stratton, Boston, MA USA, 8th ed. 2010.

[14] STEINHORST, R. K., MYERS, R. H. Classical and Modern Regression With Applications, *Journal of the American Statistical Association*, 1988, vol. 83, no. 401, p. 271.

[15] YANG, C., ZHENG, L. H., QIAO, H. T. Optimisation model of fuzzy ball fracture diverting fluid based on multiple quadratic regression method, *Materials Research Innovations*, 2014, vol. 18, pp. S2478–S2481.

[16] AL MESFER, M. K., DANISH, M., ALAM, M. M. Optimization of Performance Model of Ethyl Acetate Saponification Using Multiple Regression Analysis, *Russian Journal of Applied Chemistry*, 2018, vol. 91, no. 11, pp. 1895–1904.

[17] SABBAGH, H. A. K., ABUDAYEH, Z., ABUDOLEH, S. M., ALKRAD, J. A., HUSSEIN, M. Z., HUSSEIN-AL-ALI, S. H. Application of multiple regression analysis in optimization of metronidazole-chitosan nanoparticles, *Journal of Polymer Research*, 2019, vol. 26, no. 8.

[18] TORO, N., SALDAÑA, M., GÁLVEZ, E., CÁNOVAS, M. TRIGUEROS, E., CASTILLO, J., HERNÁNDEZ, P. C. Optimization of Parameters for the Dissolution of Mn from Manganese Nodules with the Use of Tailings in An Acid Medium, *Minerals*, 2019, vol. 9, no. 7, p. 387.

[19] MCCOY, J. T., AURET, L. Machine learning applications in minerals processing: A review, *Minerals Engineering*, 2019, vol. 132, no. November 2018, pp. 95–109.

[20] SALDAÑA, M., GONZÁLEZ, J., JELDRES, R., VILLEGAS, Á., CASTILLO, J., QUEZADA, G., TORO, N. A Stochastic Model Approach for Copper Heap Leaching through Bayesian Networks, *Metals (Basel).*, 2019, vol. 9, no. 11, p. 1198.

[21] SALDAÑA, M., TORO, N., CASTILLO, J., HERNÁNDEZ, P., NAVARRA, A. Optimization of the Heap Leaching Process through Changes in Modes of Operation and Discrete Event Simulation, *Minerals*, 2019, vol. 9, no. 7, p. 421.