THE USING OF DYNAMIC PROGRAMMING IN PURCHASING BECO ALLOYS

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Abstract

In his work, a manager will come across situations, which will require him to make easy or complicated decisions. The company management often has to take into account the sequence of decisions, of which every decision affects the following one. The tool used to find solutions to the consequential types of decision problems is called dynamic programming.

A key part of the logistics process is purchasing the necessary raw materials for production for the best possible price.

This article deals deals with the subject of optimalization of purchasing BECO alloys (an alloy of copper and beryllium) in order to achieve the highest possible conservation of financial resources, that's why the timing of purchase is a deciding factor in terms of the manager's responsibilities.

Keywords: Optimization, management, dynamic programming problem

1. INTRODUCTION

There isn't any simple model for the solving of problems of dynamic programming. That's why these problems are sorted into groups, where each one has its own formulation and resolution methods. Nevertheless, the basic approach and logic of solutions of all dynamic programming problems are the same [1].

With dynamic programming we can solve problems, which can be divided into segments according to the sequence of decisions, or we can group them into a series of smaller problems that can be dealt with first.

The analysis of dynamic programming is based on Bellman's principle of optimality, which states that: An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [2].

This principle of optimality states, that if we're starting in the normal stage, the optimal decision for the remaining stages depends only on the state of the normal stage and not on the means, by which the system reached this stage (an optimal tactic is not dependent on the decisions used in the previous stages) [3].

Contrary to most other mathematical models for dynamic programming, there isn't any standard recursive relationship. That is why it isn't possible to use a standard calculation tool (e.g. the simplex method of linear programming). However, it is possible to sort the problems of dynamic programming into ‚groups', and in so doing build a special calculation method for each one. Apart from the fact that these groups vary in structure and in calculation methods, they have a general policy of dynamic programming. These groups are the following [4]:

- Allocation problems - these problems are divided into smaller problems,
- Probability problems - these problems are divided into small problems,
- Multi Periodical problem - these problems are divided into two or more segments,
- Network problems - PERT and other networks can often be grouped among dynamic programming problems, and they are dealt with as such,
- Multiphase problems - these problems arise in production situations in the industry,

- Feedback problems - a typical area of emerging of these problems is the area of electronics, cosmonautics and car industry,
- Markov decision process.

2. MATHEMATICAL DESCRIPTION OF DYNAMIC PROGRAMMING

The relationship between the purchasing price in each stage and the optimal purchasing price (lowest) is the key to the process of dynamic programming [5]. This relationship of individual purchasing prices is called a recursive relationship [6]. For dynamic programming it is important to record the recursive relationship for each problem. Once have our equations noted, we can perform calculations of dynamic programming [7]. A recursive relationship tells us, that the optimal purchasing price in each stage for each given phase is given by the value of the best variant, where each variant includes the total purchasing price and the optimal purchasing price calculated from the previous stage.
General description

n - index for each stage tells us how many stages exist starting from the normal situation to the end of the problem,

n-1 - previous stage,

s_n - system state in the normal stage,

p - probability of the given purchasing price

c - purchasing price(cost),

E - stage,

r - revenue (expected purchasing price).

The recursive relationship in the stage n is:

$$E(r_n) = \sum p * c \tag{1}$$

Dynamic programming reduces a complex problem into a series of simpler problems. After the reduction itself, it‘s necessary to solve all the subproblems. The following methods can be used for resolving the subproblems:

- Calculation - in many cases very effective, because the amount of possible subproblems is finite and small,
- Mathematical programming - in many cases the subproblems can be solved with linear or non-linear programming,

Sequential searching - in some cases, iterative procedures can be used, in which the solution improves in every step.

3. APPLYING DYNAMIC PROGRAMMING ON THE PROBLEM OF PURCHASING BECO ALLOYS

Dynamic programming is usually associated with deterministic problems, but there are situations, where it is possible to effectively use dynamic programming for the resolving of problems of probability.

A buyer is to buy a special kind of BECO alloy (alloy of copper and beryllium), which is traded only once a week. Every week there is a 20 % chance, that the alloy will cost 8,000 €/t, 50 % chance it will cost 9,000 €/t and 30 % it will cost 10,000 €/t. He currently knows, that the alloy must be bought in the next month (within the following 4 weeks) so as to meet the production plan of the enterprise. The buyer is afraid, that if he waits too long to purchase, then a rise in price can force him to buy the alloy for a high price. On the other hand, if he buys it too soon, the future prices could decrease, and he could miss out on an opportunity to save. That is why, the time of purchase is a very delicate management task.

The decision of the buyer of ‚when to buy‘ can be perceived as a sequence of decisions. In every of those four trade weeks, the buyer must decide between two states: either to buy or to wait with the purchase. We start with the last week, and we continue back in time.

Each week is considered as one stage, this means that there are four stages available. The revenue is the expected price, which serves as a criterium.

4. SOLUTION

In this method, the first stage is the fourth. In this time, there isn‘t any option available, because the alloy wasn‘t bought yet, and so it must be bought now (**Figure 1**). The expected purchase price of the alloy is calculated in the following way:

$E(r_1) = 0.2 * 8,000 + 0.5 * 9,000 + 0.3 * 10,000 = 9,100$ €/t

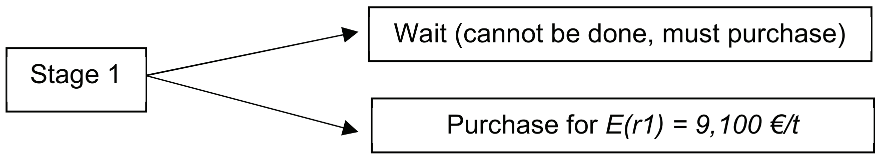

Figure 1 Decision-making in the first stage (fourth week)

In the second stage, which is the third week the buyer can buy for the prevailing price of the given week (8,000, 9,000 or 10,000 €/t) or wait until the last week. The decision is based on the following factor. If the prevailing price in the third week is greater than the expected price in the last week (9,100 €/t), then the buyer should wait until the last week. If the price is lower than the expected price in the last week, he should go along with the purchase. If the price is the same, then both options are equal. The buyer can base his decision on **Figure 2**.

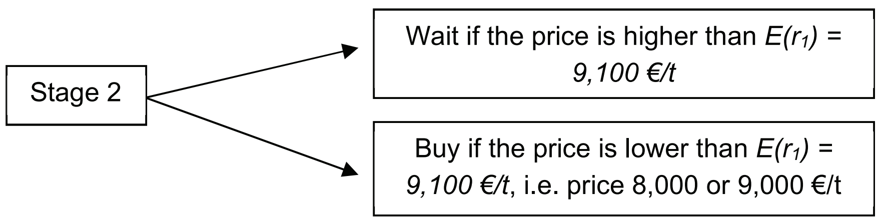

Figure 2 Decision-making in the second stage (third week)

Due to the fact, that the price in any of the stages can only have one of the three values, it is possible to calculate the expected price in the second stage:

$E(r_2) = 0.2 * 8,000 + 0.5 * 9,000 + 0.3 * 9,100 = 8,830$ €/t

In the third stage, which is the second week, the buyer can make a purchase for a price lower than $E(r_2) = 8,830$ €/t (the only acceptable price is 8,000 €) and wait, if the price gets higher. This situation is shown in **Figure 3**.

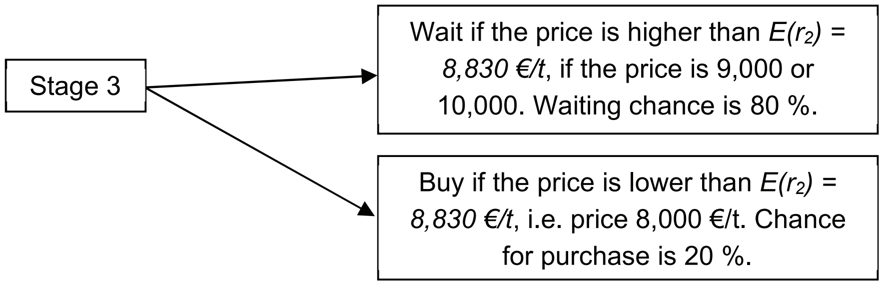

Figure 3 Decision-making in the third stage (second week)

Due to the fact, that the price in any of the stages can only have one of the three values, it is possible to calculate the expected price in the third stage:

$E(r_3) = 0.2 * 8,000 + (0.5 + 0.3) * 8,830 = 8,664$ €/t

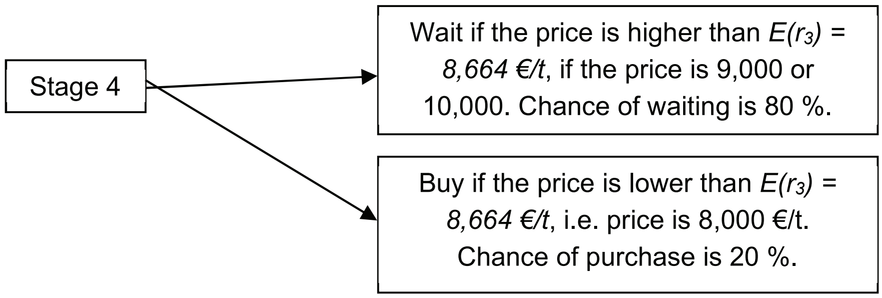

Figure 4 Decision-making in the fourth stage (first week)

In the fourth stage, which is the second week, the buyer can make a purchase for a price lower than $E(r_2)$ = *8,644 €/t* (the only acceptable price is 8,000 €) and wait, if the price gets higher. This situation is shown in **Figure 4**.

Due to the fact, that the price in any of the stages can only have one of the three values, it is possible to calculate the expected price in the fourth stage:

$E(r_4) = 0.2 * 8,000 + (0.5 + 0.3) * 8,664 = 8,531.2$ €/t

From this we can conclude the following decision-making rule for the buyer for each week, shown in **Figure 5**.

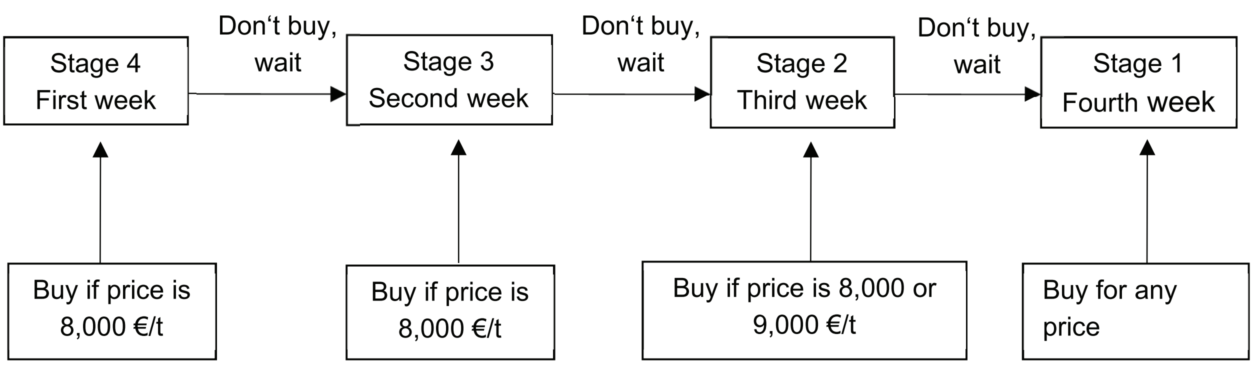

Figure 5 Decision-making rule for the buyer

5. CONCLUSIONS

If the buyer follows the optimal procedure, then he will pay the expected price 8,531.2 €/t. As can be seen on the given example, it is possible to utilize dynamic programming for probability problems. Due to the special structure of dynamic programming, it is difficult to design a standard computer program. Either a special program has to be created for each problem, or an extremely wide range of variants must be created.

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