SILVER MICRO DROP STRUCTURED TWICE AROUND THE EARTH

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Abstract

Planar micro structuring of thin metallic layers allows to achieve required surface properties of metallic layers covering bulk materials. Recently, the arrangement of micro holes or pillars placed around the primary spiral according to a phyllotactic model was presented. This deterministically aperiodic planar arrangement was used for benchmarking purposes of the e-beam writer patterning. This arrangement based on single primary spiral and a variety of derived secondary spirals has several interesting properties. One of them is a very low ratio between the area populated by individual micro elements and the length of the primary phyllotactic spiral. This paper presents analysis of the phyllotactic spiral length and the rising gradient at the spiral outer edge. The practical part of the presented work deals with the patterning of a thin silver layer deposited on the silicon or glass substrates using e-beam pattern generation, lithography techniques and related technologies. An interesting impact of the mentioned spiral properties on the e-beam writing strategies and the exposure ordering strategy are also discussed.

Keywords: Metallic thin layer, planar surface structure, phyllotactic spiral, e-beam writer

1. INTRODUCTION

This paper deals with planar metallic microstructures that are arranged according to a phyllotactic spiral model distinguished by visible *parastichy*. Planar micro structuring of thin metallic layers allows to achieve required surface properties of metallic layers covering bulk materials. Recently, the arrangement of micro holes or pillars placed around the primary spiral according to a phyllotactic model [1] was presented. This deterministically aperiodic planar arrangement was used for benchmarking purposes of the e-beam writer patterning [2]. This arrangement based on single primary spiral and a variety of derived secondary spirals has several interesting properties. This phyllotaxy based arrangement found a lot of applications in various fields, e.g. in biology [3], specific tiling arrangement [4], and magnetic resonance [5], to mention just a few of them. Also, when the seeds of the phyllotaxy model are represented by submicron optical primitives, the planar structure may have interesting light scattering [6], photoluminescence [7] or other color effects [8]. The arrangement is of high interest in the field of diffractive optically variable image devices [9]. This paper presents analysis of the primary phyllotactic spiral length. The practical part of this work deals with the patterning of a thin silver layer deposited on the silicon or glass substrates using e-beam pattern generation, lithography techniques and related technologies [10].

2. METHOD

We introduce the method by referencing the schematic image depicted in **Figure 1**. Here, one can see the primary continuous spiral of the Fermat's type with ~ 21 convolutions (**Figure 1a**). **Figure 1b** shows the same spiral plotted with a thicker line, in this case the outer part of the spiral can be no longer distinguished. **Figure 1c** and **Figure 1d** show the same spiral (in blue) but now, the coordinates of the sampled spiral seeds (phyllotaxy arrangement [1]) is appended (55 red dots). **Figure 1e** shows only the seeds from the previous figure, the primary spiral being erased. In the **Figure 1f**, the seeds are connected in a way that the scheme of secondary spirals sets appear. Two spiral sets are plotted, one set of 8 counterclockwise spirals (in black) and another set of 13 clockwise spirals (in green).

The Fermat's spiral length is deduced in Appendix A, equations (A.1-A.9). The sampled spiral has a divergent angle φ_0 as follows:

$$\varphi_0 = 2\pi/\Phi^2\,;\ \Phi = (1+\sqrt{5})/2 \tag{1}$$

Now, we should match the equations of Fermat's spiral (A.1) and the equation of Vogel's spiral -$r_{Vogel} = c\ \sqrt{(\varphi / \varphi_0)}$. Equations (2) and (3) shows an example for a divergent angle φ_0. Equations (2) and (3) put together results in relation between the Fermat's metrics factor *a* and the Vogel's metrics factor *c*.

$$r_{\varphi 0,Fermat} = a\sqrt{2\pi/\Phi^2} \tag{2}$$

$$r_{\varphi 0,Vogel} = c \tag{3}$$

$$a = c\Phi/\sqrt{2\pi} \tag{4}$$

Next, the total spiral angle of the Fermat's spiral - t_1 in the equation (A.9) - is N times the divergent angle φ_0 of the sampled spiral:

$$t_1 = N\ 2\pi/\Phi^2 \tag{5}$$

The total spiral length of the sampled Fermat's spiral is derived from the equation (A.9) by substituting *a* and t_1 from equations (4) and (5), respectively.

$$s = (4\pi/3\Phi^2)cN\sqrt{N} \tag{6}$$

3. EXPERIMENT

The previously discussed sampled spiral arrangement was performed as a planar structure with sub micron resolution. Basic parameters are selected as follows: sampled metrics factor *c* = 600 nm and the number of seeds *N* = 2.25e9 (2 250 millions). The diameter of the structure ($d_{max} = 2\ c\ \sqrt{N}$) is approximately 57 mm (~ 56.920 998 mm). The total length of this primary spiral is estimated according to equation (6) ~ 102 456. 226 439 052 km, that is *circa* twice and half larger than the Earth perimeter. The planar structure is made of thin silver layer (thickness 100 nm) sputtered on the master prepared by e-beam lithography in the electron resist (PMMA) layer spin coated on a silicon wafer.

4. RESULTS

Experiment results are demonstrated by the following figures. **Figure 2** is a metallographic microscope view of the central part of the planar structure. Secondary or derived spirals (crisscrossing spirals or *parastichy* in botany) are easy to be recognized, but the primary spiral (*cf.* **Figure 1**) is no more visible. Inlayed image shows a rank of few seeds close to the pole (center) of the arrangement. **Figure 3** is a similar view as the previous one, now the arrangement is shown in a better resolution using the SEM (scanning electron microscopy). **Figure 4** is again a light microscopy view (approximately ten times lower magnification than the one used in **Figure 2**). Here, however, the structure is not enlighten axially but from the side. One may recognize new visible patterns resulting from the light diffraction on the discussed spiral structure. Similar patterns (in fact self-similar patterns) are visible by the naked eye as it is demonstrated in **Figure 5** (camera shot of the whole spiral structure with 2.25e9 seeds, diameter of 57 mm).

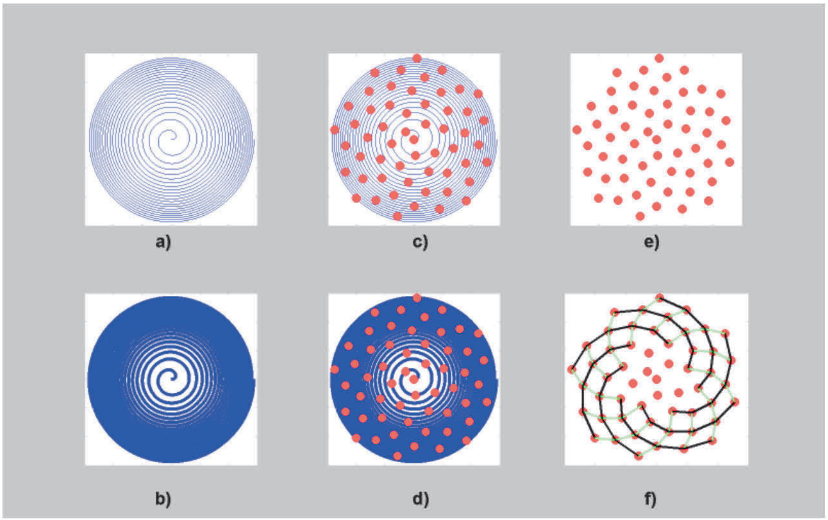

Figure 1 Schematic construction of the spiral arrangement (phyllotactic model); *see details in text*

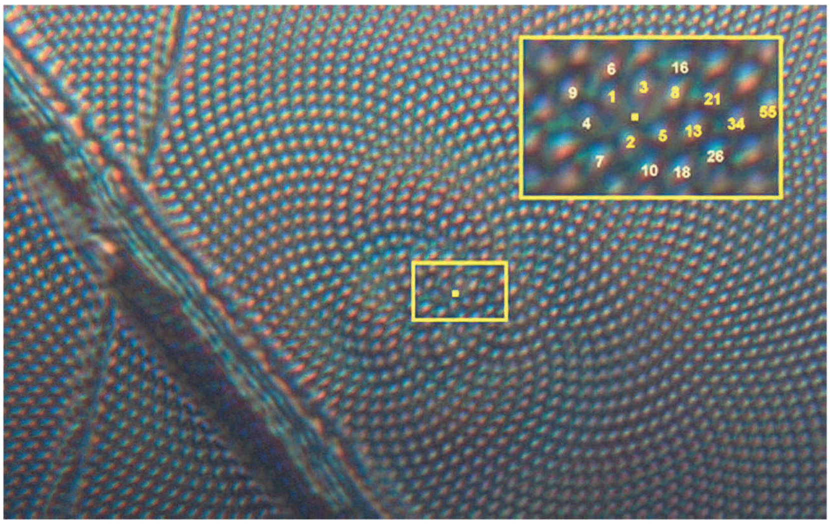

Figure 2 Central part of the spiral arrangement with the detailed view inlay; light microscope

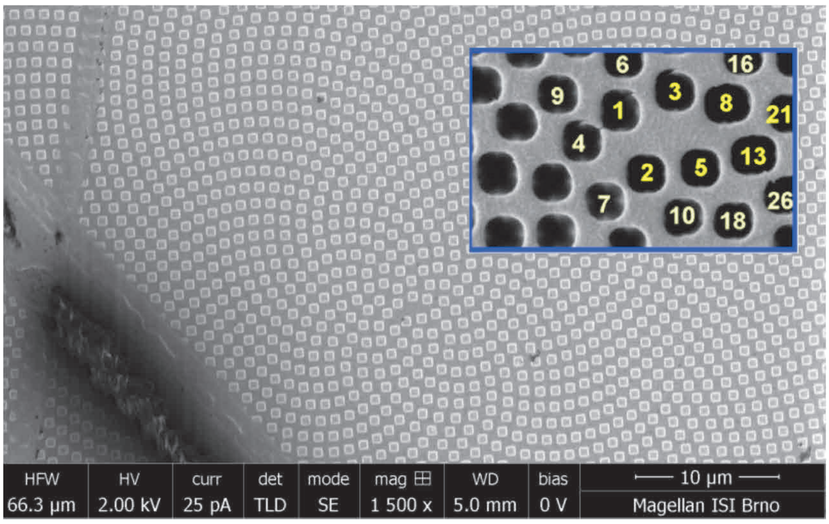

Figure 3 Central part of the spiral arrangement with the detailed view inlay; electron microscope

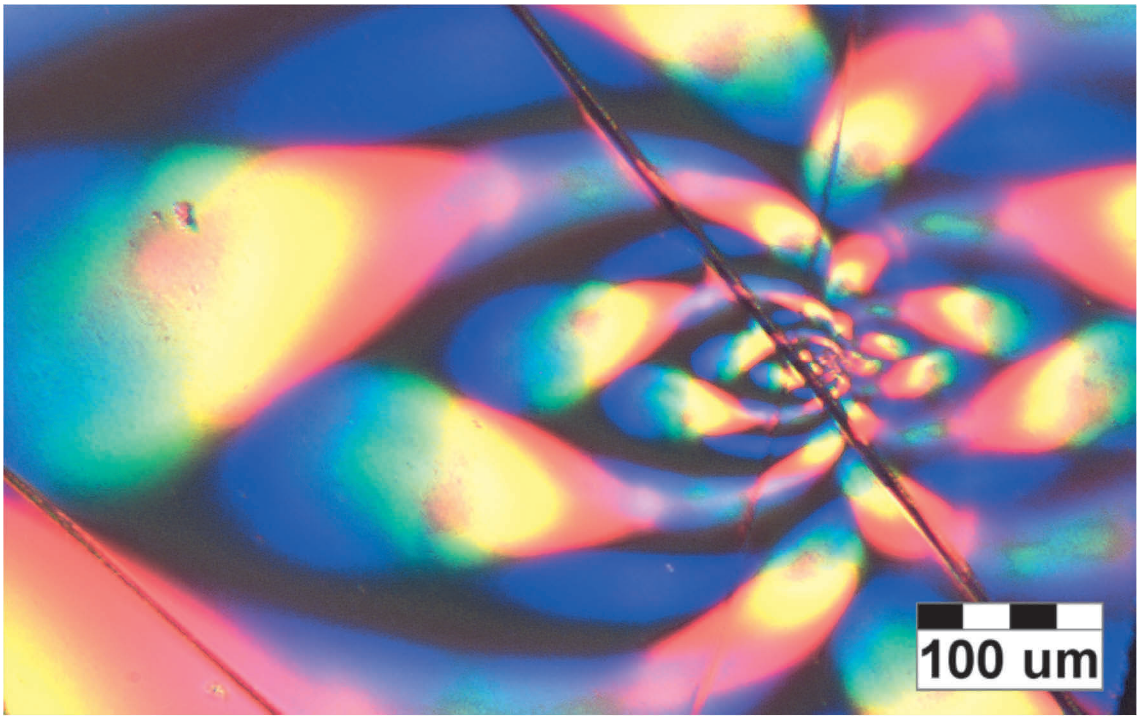

Figure 4 Central part of the spiral arrangement; light microscope, lighting from the left side

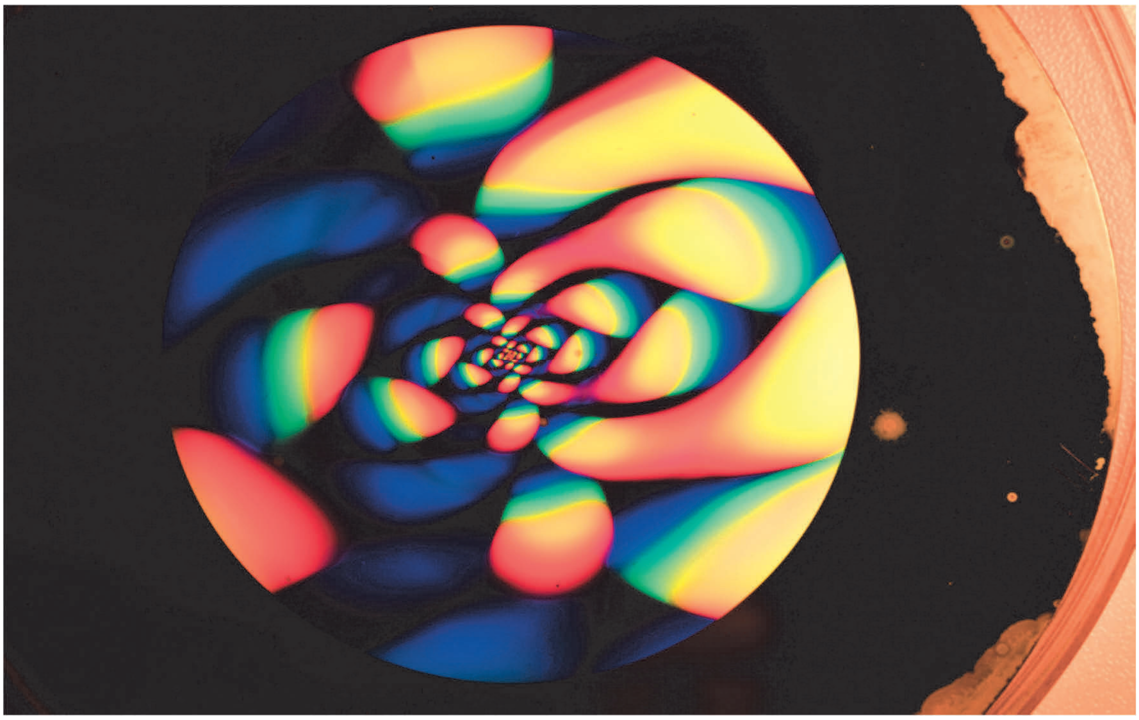

Figure 5 Complete spiral arrangement, circle diameter of 57 mm approximately

5. DISCUSSION

Silver micro drop. Given the size of the silver thin layer (outer planar structure diameter d_{max} = 57 mm, silver layer thickness t = 100 nm), the equivalent micro drop sphere has a volume ($V = t \cdot \pi \cdot d_{max}^2 / 4$) of 0.255 mm^3, from that the equivalent sphere radius ($V = \pi \cdot r_{eq}^3 \cdot 4 / 3$) r_{eq} of 0.393 mm can be derived. It might be fairly supposed that the sampled Fermat's spiral presented here is the world lengthiest spiral made from the minimum volume of material.

6. CONCLUSIONS

Planar micro structuring of thin metallic layers allows to achieve required surface properties of metallic layers covering bulk materials. The deterministically aperiodic planar arrangement originally used for benchmarking purposes of the e-beam writer patterning was exploited for the micro structuring of a thin silver layer. This arrangement is based on single primary spiral and a variety of derived secondary spirals. Presented silver planar structure in the specific arrangement achieves the spiral length of 102 thousand kilometers. This way, the power of e-beam lithography used for structuring of metallic layers was successfully demonstrated.

ACKNOWLEDGEMENT

The research was partially supported by the TACR (projects TE01020233 and TG03010046), by MEYS CR (project LO1212), its infrastructure by MEYS CR and EC (CZ.1.05/2.1.00/01.0017) and by CAS (RVO:68081731).

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APPENDIX A: ARC LENGTH OF FERMAT'S SPIRAL

We start from the basic description of the Fermat's spiral in the following form:

$$r = a\sqrt{\varphi} = g(\varphi), \qquad \varphi \in \langle 0, \varphi_1 \rangle \tag{A.1}$$

A particular formula for the arch length in this case is

$$s = \int_0^{\varphi_1} \sqrt{g^2(\varphi) + [g'(\varphi)]^2} d\varphi = \frac{a}{2} \int_0^{\varphi_1} \sqrt{\frac{1+4\varphi^2}{\varphi}} d\varphi. \tag{A.2}$$

Using polar coordinates yield functions

$$x = r \cdot \cos t \, , y = r \cdot \sin t \tag{A.3}$$

giving $\varphi = t$ and $r = a\sqrt{t}$ for the functions

$$x(t) = a\sqrt{t} \cdot \cos t \, , y(t) = a\sqrt{t} \cdot \sin t \tag{A.4}$$

results in

$$s = \int_0^{\varphi_1} \sqrt{(x'(t)^2 + y'(t)^2} dt = \frac{a}{2} \int_0^{\varphi_1} \sqrt{\frac{1+4t^2}{t}} dt, \tag{A.5}$$

which corresponds to the formula (A.1) stated in the beginning, $r = g(\varphi)$.

Therefore, we would like to solve the binomial integral

$$s = \frac{a}{2} \int_0^{\varphi_1} \sqrt{\frac{1+4t^2}{t}} dt = \frac{a}{2} \int_0^{\varphi_1} t^{-\frac{1}{2}} (1+4t^2)^{\frac{1}{2}} dt = \frac{a}{2} \int_\alpha^\beta t^m \, (a+bt^n)^p dt \tag{A.6}$$

for $m = -\frac{1}{2}, n = 2, p = \frac{1}{2}$.

Since none of the numbers $\frac{m+1}{n} = \frac{1}{4}, \frac{m+1}{n} + p = \frac{3}{4}$ is a whole number, the standard substitutions cannot be used. If we use for example a substitution, where $t = p^2$ for $p \geq 0, dt = 2pdp, p \in < 0, \sqrt{t_1} >$, we get

$$s = \int_0^{\sqrt{t_1}} \sqrt{1+4p^4} dp, \tag{A.7}$$

which is a complicated elliptical integral that is solved using special mathematic functions.

Instead of using the classical Taylor's expansion, a more precise estimation may be used, where

$$\sqrt{1+4p^4} \sim \sqrt{4p^4} = 2p^2 \tag{A.8}$$

Using this estimation within the integral in the equation (A.7), we get a formula for the spiral length

$$s = a\sqrt{t_1} \cdot \frac{2}{3} t_1. \tag{A.9}$$

The final equation (A.9) is very handy to use and also it is adequately precise, particularly when the Fermat's spiral becomes very large.