

**STATISTICAL ANALYSIS OF THE 3D SCAN RESULTS OF WAX MODEL KITS AND MULTI-LAYERED CERAMIC MOULDS FOR INVESTMENT CASTINGS USING IN AVIATION WITH THE ADVANCED SIX SIGMA TOOLS**

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**Abstract**

Due to the specific requirements for monitoring processes and products in the aircraft industry, non-destructive testing (NDT) is increasingly used in combination with advanced statistical analysis.

The paper presents the results of research aimed at determining the quality of wax models and multi-layered ceramic forms, used for precise casting of critical parts of aircraft engines. Studies on the shape and dimensions of wax models and ceramic moulds were made by 3D scanning using the GOM ATOS CORE 200 scanner. A map of dimensional deviations, created as a result of comparison of scanned wax models with CAD models, was created. The thickness of the moulds was determined by comparing the dimensions obtained by scanning 3D wax models and ceramic forms.

The obtained measurements were then subjected to advanced statistical processing using selected Six Sigma tools. The analysis was carried out in several separate areas of both wax models and forms. The basis of the analysis were the results of measuring 30 points in each of the measurement zones of the blades and moulds models. The consistency of the results with the normal distribution based on the empirical cumulative distributor was verified. The Anderson-Darling test was adopted based on the weighted distance between empirical and theoretical counselor.

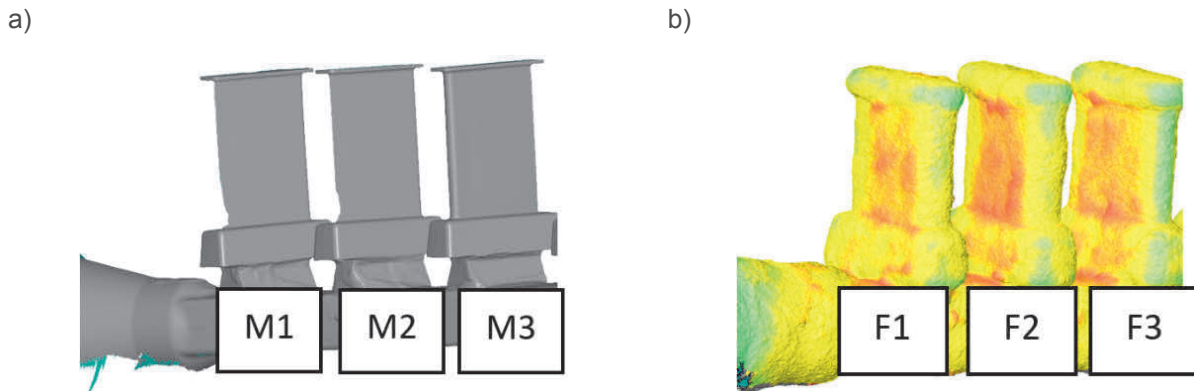
**Keywords:** Six Sigma, investment casting, statistics, 3D scanning

**1. INTRODUCTION**

In the aviation industry, due to the specified requirements for monitoring processes and products, non-destructive testing methods correlated with advanced statistical analysis are increasingly used. One of the innovative and very accurate way of measuring is 3D scanning [1] involving the use of a 3D scanner to take a series of images saved in an .stl file and then processing them with the use of dedicated software for scans enabling determination of shape and dimensions at each point of the tested object. The application of this method in aviation has been presented in the article [2]. A precise statistical analysis of the results obtained is possible, among others, using the mathematical apparatus used in the Six Sigma methodology [3]. The paper presents the results of tests of model sets and multi-layered ceramic forms used, among others, for the production of critical parts of aircraft engines in the form of turbine blades produced using the lost wax method [4]. On 3D model sets and ceramic molds 3D measurements were made and the results were subjected to detailed statistical analysis in 9 zones characteristic for turbine blades. The results allowed to obtain information on the possible shrinkage of wax models and the thickness of the ceramic mold.

**2. METHODOLOGY**

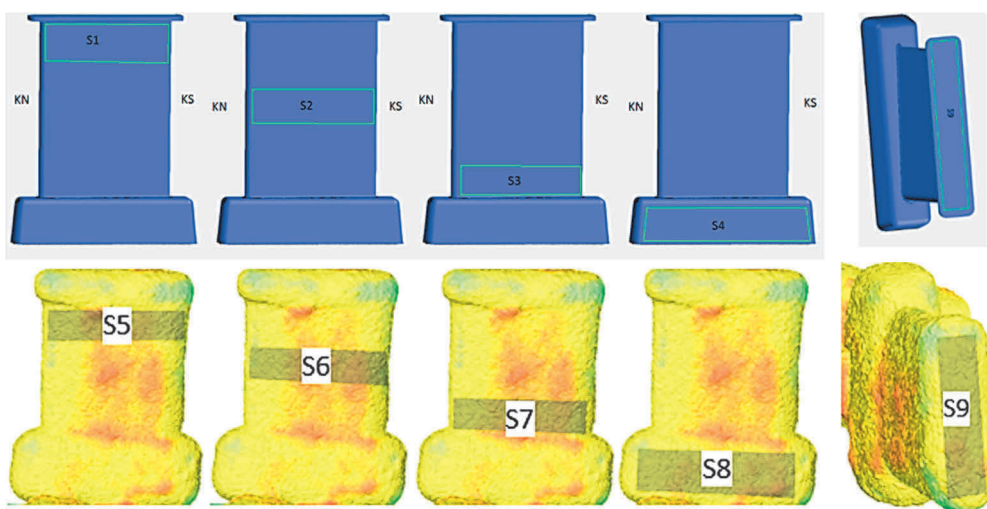
The primary objective of the research was the dimensional characterization of wax blades (M1, M2, M3 - **Figure 1a**) and multi-layered ceramic forms (F1, F2, F3 - **Figure 1b**) based on 3D scanning results obtained using the Atom Core scanner from GOM. In the case of wax models, the results of the scan were compared with the CAD models, resulting in dimensional deviations resulting from, for example, a wax shrinkage or non-fill (**Figure 1a**). In the case of ceramic molds, the results of the scan were aligned with the results of the wax model scanning, resulting in the thickness of the mold (**Figure 1b**).



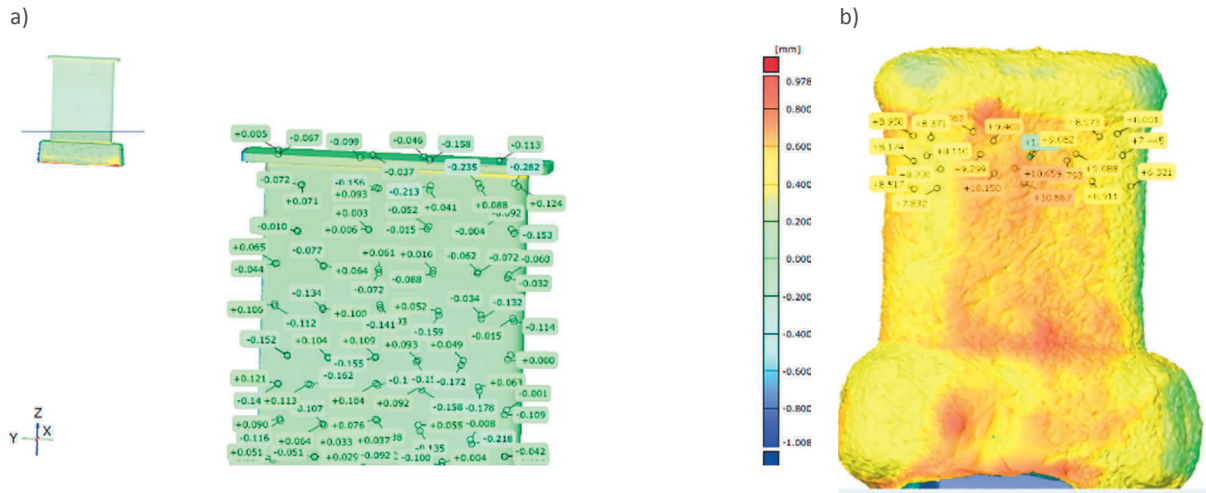
**Figure 1** 3D scan of a wax model kit (a) and a ceramic mold (b)

In the model of the blade and mold, 9 measurement zones S1 - S9 were separated (**Figure 2**). The basis of the analysis were the results of measuring 30 points in each of the measuring zones of blade and mold models (**Figure 3**). The consistency of the results with the normal distribution was verified based on the empirical cumulative distribution function. The Anderson-Darling test was adopted based on the weighted distance between empirical and theoretical distribution functions. Results with a large deviation from the normal distribution were discarded. It was found that the remaining results correspond to the normal distribution and provide a good basis for statistical analysis.

The mean value, standard deviation, the minimum and maximum value of the dimensional deviation from the nominal values were determined for each measuring field of the model and the mold.



**Figure 2** Measuring zones of the wax model: S1 - S4, S9 and ceramic mold S5-S9, KS - trailing edge, KN - leading edge



**Figure 3** Sample results image obtained from scanning a wax model (a) and a ceramic mold (b)

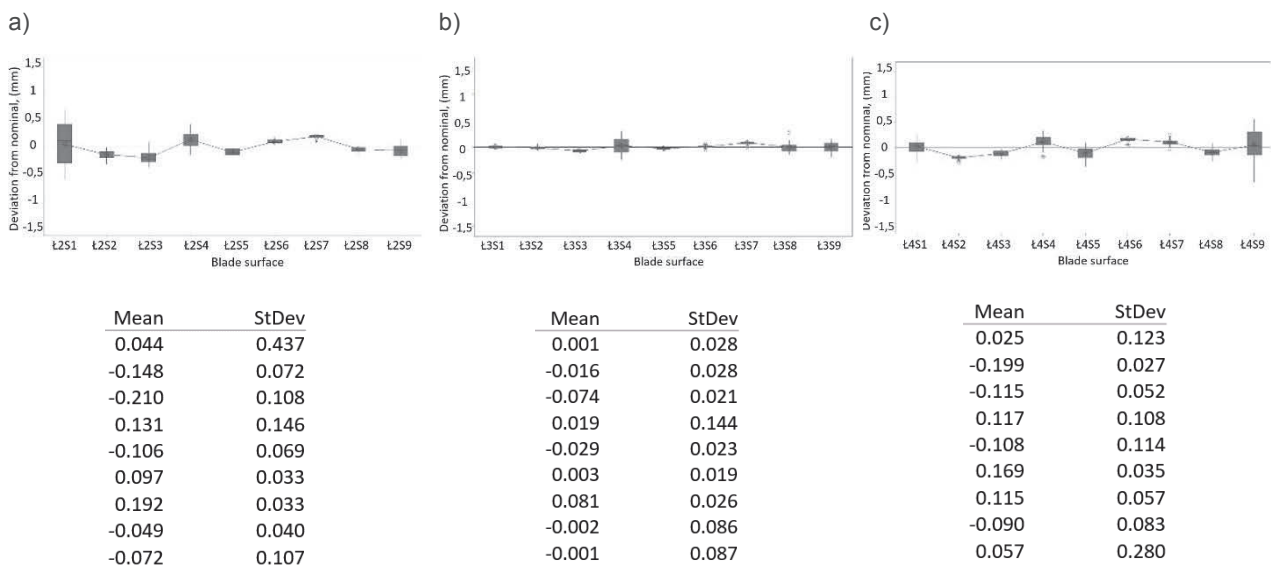
The aim of the comparative statistical analysis was to determine the degree of dependence of the obtained results from the measurement zone of the blade model and the blade form (S1 - S9). The comparative analysis was conducted on the basis of the nonparametric test of the one-way analysis of variance (ANOVA) at the level of significance  $\alpha = 0.05$ .

### 3. RESULTS

On the basis of the analysis of the measurement results, dimensional characterization of wax models and ceramic forms in the designated measurement zones S1 - S9 was made. The results of the analysis are the basis for further dimensional analysis of the casting process after the casting.

#### 3.1. The results of research on wax models

**Figure 4** shows the results of mean value and standard deviation for M1, M2 and M3 in zones S1 - S9.



**Figure 4** Box plots of mean value and standard deviation for the wax model of the blade M1 (a), M2 (b) and M3 (c)

It was found that the greatest values of deviation from the nominal value are characteristic for zones S4 and S8 (blade root)  $\pm 0.996$  mm. S9 zone (blade shelf) characterizes the maximum value of deviation  $\pm 0.562$  mm. While blade surface (S1 - S3 and S5 - S7) is characterized by a large spread of results. There are deviation values even up to 0.997 mm but there are few of them - over 80 % of the results do not exceed the deviation of 0.583 mm. Graphical interpretation of measurement results (**Figure 5**) indicates that there are no differences between the populated populations. However, statistical tests have shown that the differences are significant. As a result of the statistical analysis, it was found that there are no grounds to accept the hypothesis  $H_0$  on the significance level  $\alpha = 0.05$  for all hypotheses (1) - (3).

$$H_0: (R) = (Sz) \tag{1}$$

$$H_0: S^2(C) = S^2(G) = S^2(P) \tag{2}$$

$$H_0: S^2(S1) = S^2(S2) = \dots = S^2(S9) \tag{3}$$

where:

$H_0$  – null hypothesis

$S$  - variance

$R$  – parallel set

$Sz$  – serial set

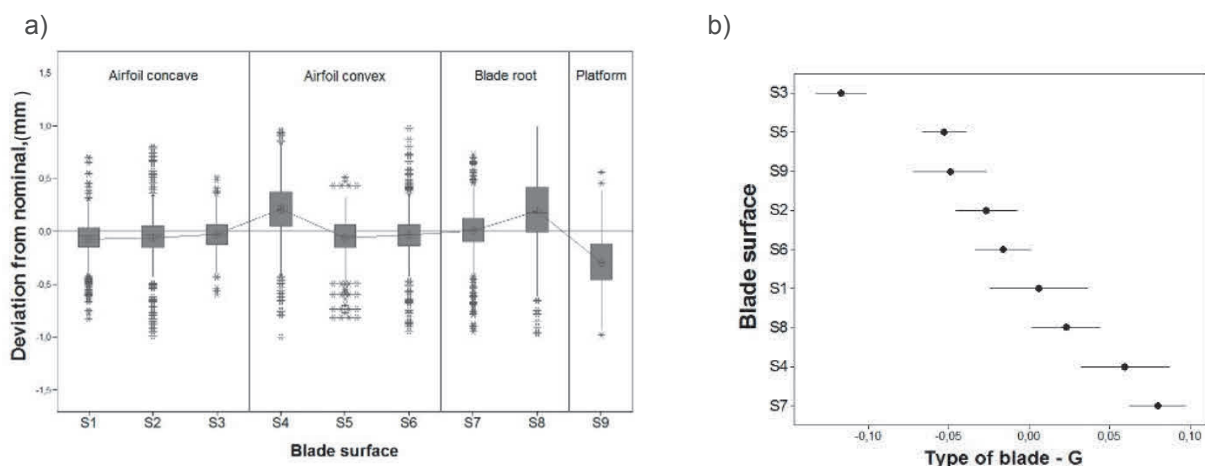
$C$  – thin non-profiled blade

$G$  - thick non-profiled blade

$P$  - profiled blade

$S1 - S9$  – zones defined on **Figure 2**

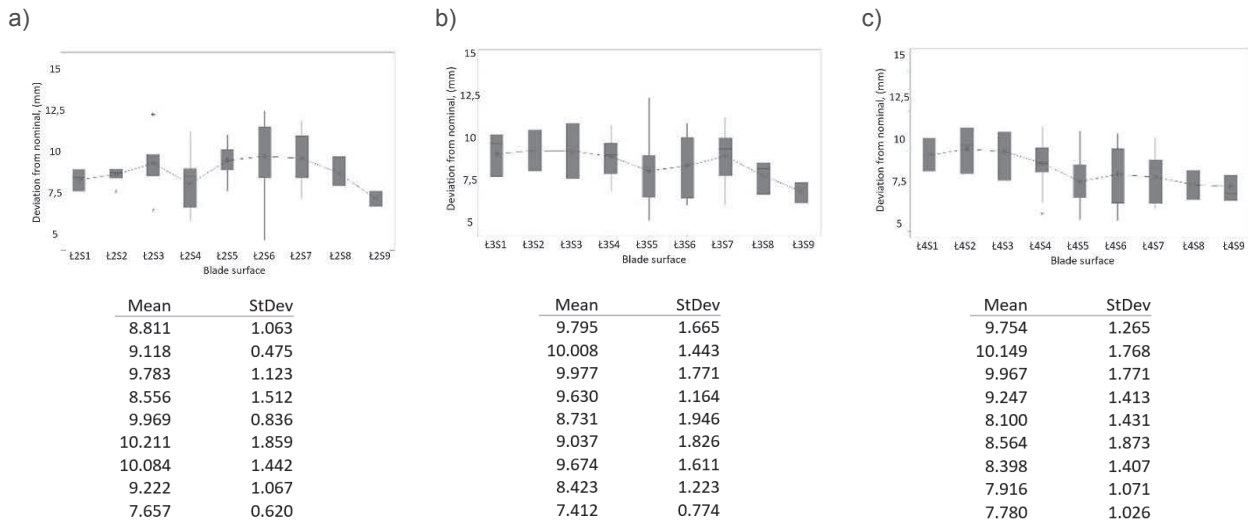
Due to the large differences in the variance between zones describing surface, root and shelf of the blade (**Figure 6**), it was assumed that these zones can be treated integrally and analyzed separately in further research after the mold creation and casting.



**Figure 5** The deviation from the nominal value for the measurement zones of the blades model (a) and difference in variance deviation value for blade measurement zones (ANOVA analysis) (b)

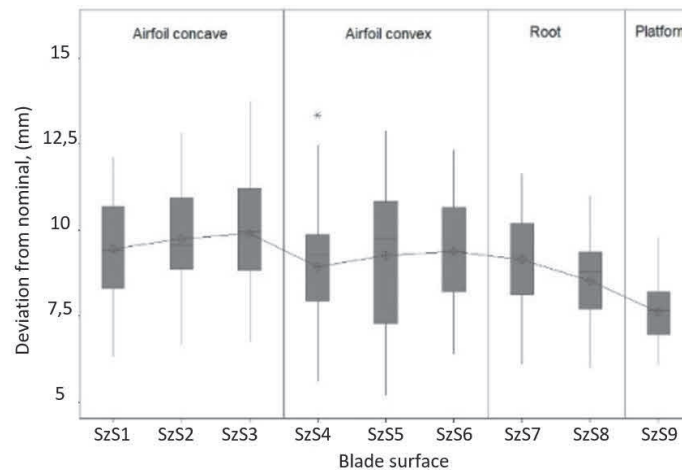
### 3.2. The results of the ceramic form testing

The roughness / graininess of the surface of the ceramic mold is high and results in a large spread of results depending on the position of the measuring point - in the cavity of the mold or on the ceramic grain. The value of the scatter of results is large and therefore varies from 2.2 to 9.3 mm. Hence, some sets of measurement results do not fulfill the normal distribution requirement. However, due to the specifics of the casting surface of the ceramic form they have been included in further calculations.



**Figure 6** Box plots of the mean value and standard deviation for the mold F1 (a), F2 (b) and F3 (c)

On the basis of the analysis of the measurement results, dimensional characterization of ceramic forms in the designated measurement zones S1 - S9 was made. It was found that the greater spread of the range of deviation from the nominal value is 5.20 / 13.74 (**Figure 7**).



**Figure 7** The deviation from the nominal value for the measurement zones of the blades mold

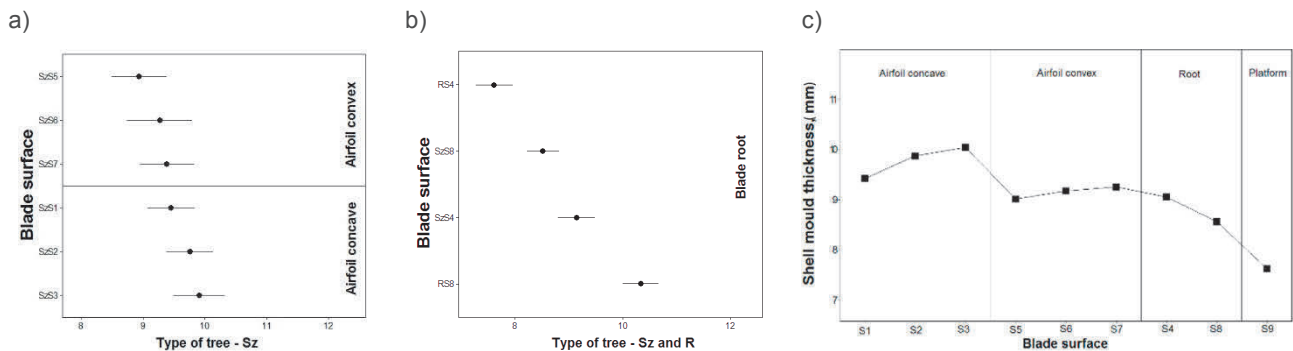
As a result of the statistical analysis, it was found that the value of the test statistic p for all hypotheses (4-5) is <0.05. There are no grounds for accepting the  $H_0$  hypothesis at the significance level of  $\alpha = 0.05$ . Hence, it was assumed that the type of the casting set and especially the side of the blade surface (suction side shell vs pressure side shell) significantly influence the results of the deviation from the nominal value of ceramic molds (**Figure 8 a, b**). Higher values of deviations in the serial model set occur from the suction side.

$$H_0: (R) = (Sz) \tag{4}$$

$$H_0: (S1) = (S2) = \dots = (S9) \tag{5}$$

with notation as in equations (1) - (3).

The reference of the results of dimensional analysis of the ceramic mold to the results of the measurement analysis of wax models allows to determine the thickness of the casting mold. The average thickness of the mold is 9.12 mm. However, the analysis of the casting mold thickness results in the designated zones confirmed the trends observed for the results of deviation from the nominal value (**Figure 8c**).



**Figure 8** Difference in mean deviations for the blade mold measurement zones (ANOVA analysis); blade surface (a), blade root (b). Thickness of the casting mold - average values (c)

The largest mold thickness is for the S1-S3 area and is approximately 10 mm. For the S4-S8 area, the thickness is 9 mm and in the area of S9 it is approx. 8 mm. The results indicate the variation in the thickness of the mold, which can consequently result in the different heat dissipation of the mold after its flooding and the casting defects of the blades.

#### 4. CONCLUSION

As a result of the analysis of the obtained results, the following conclusions were formulated:

- The distribution of deviations from the model of the blade model in the range of  $\pm 1.0$  mm will allow a detailed assessment of the applied measurement method.
- The results from measurement zones S1-S9 separated on the blade should be analyzed separately due to the existence of significant differences already at the stage of measuring the wax model.
- Also for the mold, the results obtained from individual zones differ significantly which confirms that analyzing them separately was necessary.
- The differences in the thickness of the form in the zones described are most likely caused by the position in which the form drying.
- The differences in the thickness of the mold are so large that they can have a significant impact on the way the metal solidify inside.

#### ACKNOWLEDGEMENTS

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