ATTRIBUTES CONTROL CHARTS AS A TOOL OF RETROSPECTIVE ANALYSIS

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Abstract

The paper deals with attributes control charts, especially with the control charts for fraction nonconforming, and their use in retrospective analysis of a process. Several forms of these charts are applied in a case study. The study concerns the occurrence of defects, especially the deformations on hot-rolled wire manufactured on the wire rod mill at Arcelor Mittal Ostrava a.s. Number of wire coils with defects is of interest. *P* charts are used to identify subgroups with an inflated fraction nonconforming that could distort results of evaluating differences in fraction nonconforming between steel grades, hot-rolling temperatures, and wire diameters. In addition, it is shown how to modify the traditional *p* chart in the case when a substantial part of the points fall outside control limits due to large subgroup sizes.

Keywords: Outliers, *p* chart, Laney's chart, hot-rolled wire, deformations

1. INTRODUCTION

In the present highly competitive business environment, it is necessary to monitor and reduce the occurrence of nonconformities in production processes. During production of hot-rolled wire, nonconformities of two kinds occur: wrong dimensions of the produced wire and defects of surface and shape, among which deformations are most common. Diameter and ovality measurements are performed using operational or special gauges or specially designed control templates, nonconformities of the second kind are detected using visual inspection. As a part of the process analysis, intensities of nonconformities or proportions of nonconforming units under different operational conditions may be compared.

Frequent practice in comparing proportions of nonconforming units at different stages of the process quality control is to pool nonconforming items from all subgroups within a stage and calculate the fraction nonconforming as a ratio of the total number of nonconforming items and the total number of inspected items in this stage. Statistical tests assume that population proportions are compared using random samples from the populations compared. So that the grouped data within a stage can be considered a random sample, the process homogeneity has to be verified. Attributes control charts described in [1] and in publications dealing with statistical process control, see for example [2] and a number of others, can serve as a simple tool. Different subgroup sizes and especially large subgroup sizes require a certain adjustment of the traditional control limits [3].

In this paper, control charts for fraction nonconforming are applied to identify subgroups with particularly high proportion of wire coils with deformations before the fraction nonconforming at different temperatures, steel grades and wire dimensions is examined.

2. CONTROL CHART FOR FRACTION NONCONFORMING

When items in a production process are classified as conforming or nonconforming ones, the fraction nonconforming π is of interest. It is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population. The fraction nonconforming may be tracked on a control chart. Data are collected in subgroups which may be of varying sizes and the sample fraction nonconforming p_j for $j = 1, 2, ..., k$, defined as the ratio of the number of nonconforming items x_j to the sample size n_j is plotted. It is assumed that the probability that any item will not conform to specification equals π, and that each item is independent of its predecessors. The construction of the control limits of the *p* chart is based on the assumption that the number of nonconforming items *X* in a sample of size *n* has the binomial distribution with parameters *n* and π. Its mean is E(*X*) = $n\pi$ and the variance is D(X) = $n\pi(1 - \pi)$.

Then the sample fraction nonconforming *X*/*n* has the mean and variance

$$E(X/n) = \pi \qquad D(X/n) = \frac{\pi(1-\pi)}{n} \tag{1}$$

Usually π is estimated by $\overline{p} = \sum_{j=1}^{k} x_j / \sum_{j=1}^{k} n_j$ .The control limits of the *p* chart are based on the normal approximation of a binomial distribution. The centre line in the *p* chart corresponds to $\overline{p}$ and the three-sigma limits are [1,2]

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{2}$$

Differently from a normal distribution, the binomial distribution is generally asymmetric, which may lead to some undesirable properties of the *p* chart. So that the approximation is good enough, sample sizes should be sufficiently large. Some recommendations can be found in literature, such as $n\pi(1-\pi) > 5$ and $0.1 \le \pi \le 0.9$, or $n\pi(1-\pi) > 25$, see [4], or $n\pi > 5$ and $n(1-\pi) > 5$, see [5].

According to [6], the sample size should be large enough so that the probability of detecting a shift of magnitude δ from a specified value π_0 is around 0.5. Therefore $n = (3/\delta)^2 \pi_0 (1-\pi_0)$.

If a 100 % inspection of the process output is performed, sample sizes usually differ. Unless the differences are small, either variable control limits are constructed for each sample size n_j,

$$LCL_j = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n_j}} \qquad UCL_j = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n_j}} \tag{3}$$

or a standardized control chart with the plotted values z_j is used, where [1,2]

$$z_j = \frac{p_j - \overline{p}}{\hat{\sigma}_{p_j}}, \qquad \hat{\sigma}_{p_j} = \sqrt{\frac{\overline{p}(1-\overline{p})}{n_j}} \tag{4}$$

Under the assumption $p_j \sim \mathrm{Bi}(n_j, \pi)$ variable z_j is normally distributed, $z_j \sim N(0,1)$. Therefore the standardized chart has the centre line at zero and control limits at ± 3 .

When sample sizes are large, the control limits given by (2) or (4) are too narrow and the *p* chart is very sensitive to changes in π. The above charts are unusable due to the high number of points falling outside the limits. Laney [3] suggested the use of the chart for individuals used in control for variables where z-scores according to (5) are plotted. Differently from the standardized *p* chart, the standard deviation σ_z of z-scores is not assumed to be equal to 1 but it is estimated in the same way as in the charts for individuals, i.e. using moving ranges

$$R'_j = | z_j - z_{j-1} |, \quad j = 2, ..., k \tag{5}$$

The estimated standard deviation is

$$\hat{\sigma}_z = \overline{R}' / 1.128 \tag{6}$$

where $\overline{R}' = \frac{1}{k-1} \sum_{j=2}^{k} R'_j$.

To detect overdispersion in attribute data when the observed variability is greater than would be expected under binomial assumptions, the graphical method introduced in [7] and implemented for example in Minitab can be used. The method involves transforming the data and using the normal probability plot. If the ratio of the observed variation and the expected variation exceeds the empirically determined 95 % upper limit, the use of the Laney's chart is appropriate. Details are given in [7].

3. RETROSPECTIVE ANALYSIS

In most control charts the sample fraction nonconforming is plotted over time to determine whether a process is stable. However, the *p* chart can have an additional use; it may serve as a tool of exploratory analysis to identify outliers before a subsequent analysis is performed. For example, if the fraction nonconforming under different operating conditions in short-run processes is to be compared and data from individual runs are grouped, it is not advisable to compare the run averages and ignore the within-group variation of the fraction nonconforming.

Under the assumption that all n_j are sufficiently large, the proportions in K populations can be compared using the chi-square test with the statistic [8]

$$\chi^2 = \frac{1}{p(1-p)} \sum_{j=1}^{K} n_j p_j^2 - \frac{1}{N} (\sum_{j=1}^{K} n_j p_j)^2 \tag{7}$$

and the critical region $\chi^2 \geq \chi^2_{1-\alpha}(k-1)$. The equivalent form of the statistic for K = 2 is

$$u = \frac{p_1 - p_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{8}$$

with the critical region $|u| \geq u_{1-\alpha}$. Another test statistic based on the range of the arcsine transformations of the observed proportions is introduced in [9].

Different process runs may be characterized by combination of several factors and effects of these factors on the fraction nonconforming may be of interest. In such cases ANOVA can be used. F-test in ANOVA is based on the assumption of normal distribution and homogeneity of variances. If applied on subgroup proportions, departures from these assumptions may be large. The possible solution is the use of a transformation [2,5]. The most simple one is $\arcsin\sqrt{p}$.

4. CASE STUDY

Data come from daily reports summarizing the results of monitoring nonconformities in the production of hot-rolled wire at Arcelor Mittal Ostrava a.s. According to customer requirements, different parameters of the wire rolling process are set. To ensure required mechanical properties, steel grades C36R or C39 according to DIN

59 110 are used. Another important parameter is a hot rolling temperature. The higher temperature (1,000 °C) is used to provide better mechanical properties of rolled wire, the lower temperature (820 °C) is used when a fine steel microstructure is preferred. The orders with two most common values of a diameter were chosen: 5.5 mm and 9 mm.

The wire is winded into coils weighing from 2 to 2.9 t. The coils from the same heat form a subgroup. Subgroups vary in size due to different volumes of production orders and 100 % inspection. Deformations of the wire influence the smooth unwinding of the coils and its further processing. Wire coils with deformations are considered nonconforming units and therefore the *p* chart for fraction nonconforming was applied.

The different process stages are characterized by three parameters: the steel grade, wire dimension and temperature. Combinations of three process parameters are referred to as experimental conditions and they are described in **Table 1**.

Table 1 Experimental conditions (stages in control charting)

Stage	1	2	3	4	5	6	7	8
Temperature (°C)	1,000	820	1,000	820	1,000	820	1,000	820
Steel grade	C36R	C36R	C39R	C39R	C36R	C36R	C39R	C39R
Dimension	D5.5	D5.5	D5.5	D5.5	D9	D9	D9	D9
Number of heats	97	33	18	24	69	36	49	63
Total number of coils	12,656	3,825	1,372	2,062	8,519	4,696	5,530	7,773

First the variation between heats within the same experimental conditions was examined through a *p* chart. The *p* chart was arranged in stages - subgroups belonging to the same experimental conditions formed a stage (see **Figure 1**) with its own centre line. Due to large differences between subgroup sizes the charts with variable limits had to be constructed.

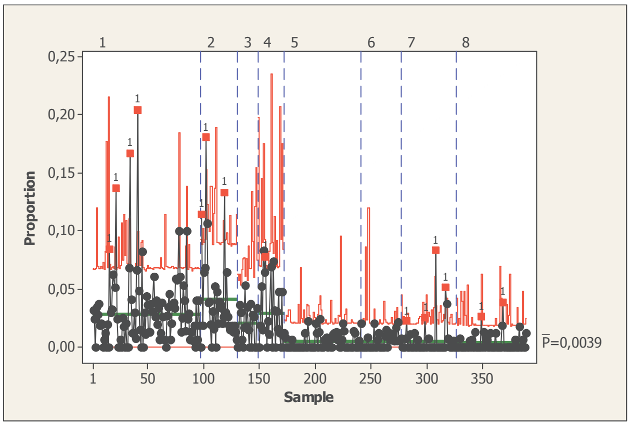

Figure 1 *P* chart

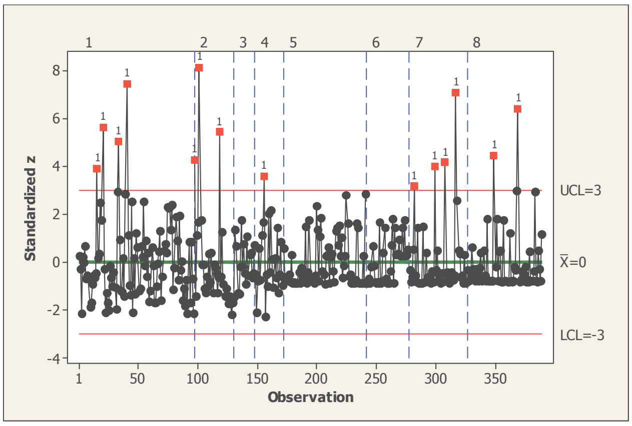

Figure 2 Standardized *p* chart

The chart in **Figure 1** clearly indicates the difference between the fraction nonconforming in D5.5 (stages 1 to 4) and D9 (stages 5 to 8). The existence of differences between two steel grades or two temperatures is not obvious and must be investigated using a statistical test. However, the chart above all revealed that in some stages several subgroups have an extremely large fraction nonconforming. For a better visualization of outliers, a standardized version of the *p* chart was used (see **Figure 2**) and subsequently the subgroups exceeding the upper control limit were removed.

Subgroup proportions p_j were transformed using the arcsine transformation mentioned above and 3-factor ANOVA was applied. The results are shown in **Table 2**. Only the rows for main effects are displayed; no interaction was significant (the least P-value was 0.4397).

Table 2 ANOVA on transformed data, main effects

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A: Temperature	0.0107	1	0.0107	2.40	0.1225
B: Steel grade	0.0512	1	0.0512	11.51	0.0008
C: Dimension	0.7885	1	0.7885	177.08	0.0000

It can be seen that the main effects of the steel grade and diameter are significant, whereas no significant difference in fraction nonconforming at different temperatures has been proved.

If we want to investigate only the effect of temperature assuming that the existence of the effects of steel grade and diameter is known, the u-test according to (8) should be used separately for each experimental conditions. The results are in **Table 3**.

Table 3 Comparison of fraction nonconforming at different temperatures

Compared stages	**1 - 2**	**3 - 4**	**5 - 6**	**7 - 8**
u	-0.0946	-0.1003	0.0077	-0.0666
P-value	0.9246	0.9201	0.9939	0.9489

It can be concluded, that the differences in the fraction nonconforming under the two temperatures are not significant (all P-values much higher than 0.05). Without removing outliers the difference between stages 1 and 2 would appear significant with u = -3.9817 and the corresponding p-value less than 0.001.

5. EXAMPLE OF OVERDISPERSION IN DATA

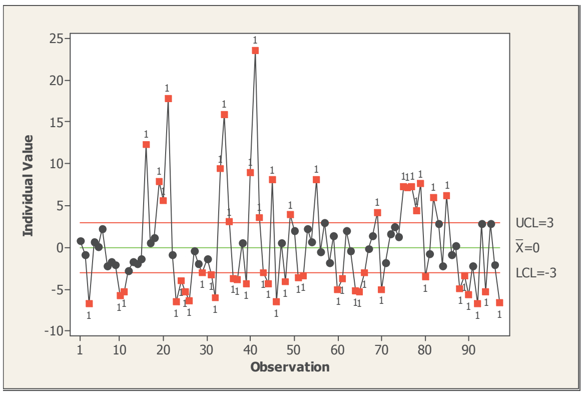

Figure 3 Standardized *p* chart

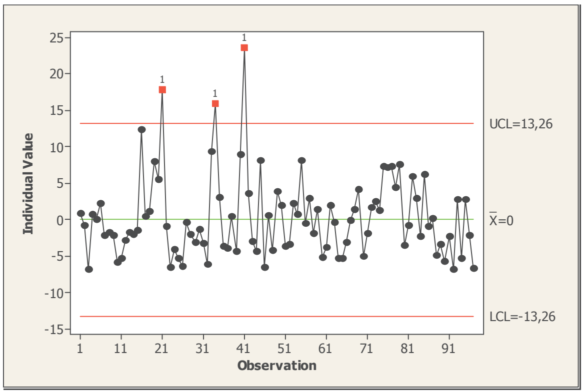

Figure 4 Standardized Laney's chart

To point out the problem occurring with large sizes of subgroups, let us consider the same fractions nonconforming as before, but now we assume that the subgroup sizes were ten times larger. For greater clarity we will consider only the first stage (see **Table 1**). The standardized *p* chart is shown in **Figure 3**. The control limits are narrow due to large subgroup sizes and they are exceeded by many points. Such a control chart is unusable because minor deviation from the average fraction nonconforming cannot be considered as a reason to intervene in the process or treated as outliers in the subsequent analysis. After checking *p* chart assumptions in Minitab, the ratio of observed variation to expected variation is equal to 154.7 %, which is higher than the 95 % upper limit of 134.7 % and therefore the Laney's chart for *z* with the estimated standard deviation $\hat{\sigma}_z = 4.419$ is used (see **Figure 4**). Now only three points representing groups no. 21, 34, and 41 fall outside the control limits. These points should be treated as outliers in a subsequent analysis.

6. CONCLUSION

The main goal of the paper was to show the usage of a *p* chart or its modification as a tool of an exploratory data analysis. The paper demonstrates the need to verify assumptions before a statistical test is performed and illustrates how the occurrence of outliers may distort the results of hypothesis tests. Without removing heats with the inflated fraction nonconforming based on the *p* chart, the cause of the significant difference between the fractions nonconforming at the two temperatures would be investigated in vain.

Based on the ANOVA applied on the transformed data, differences between the fraction nonconforming at two steel grades and two wire diameters are significant, whereas the different temperatures do not appear to affect the fraction nonconforming. The latter conclusion is confirmed by the u-test, which is more appropriate in this study due to the better fulfilment of its assumptions.

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