APPLICATION COBB-DOUGLAS PRODUCTION FUNCTION FOR ANALYSIS OF PRODUCTION IN POLISH STEEL INDUSTRY

GAJDZIK Bożena[1]

[1]*Silesian University of Technology, Faculty of Materials Engineering and Metallurgy, Katowice, Poland, EU*
bozena.gajdzik@polsl.pl

Abstract

The article presents the use of Cobb-Douglas production function (C-D model) to analyse the impact of two basic factors of production: capital and labour of its size in Polish steel industry. Paul Cobb and Charles Douglas wrote the function as: $P = a \cdot L^{\alpha} \cdot C^{1-\alpha}$ (where P is the production of L - labour, C - capital). In this century the function was popularized in the neoclassical model of the company. Currently, the C-D function is used to assess the scale of production in economy, industry and company. In the paper the function (changed by David Durand): $P = a \cdot L^{\alpha} \cdot C^{\beta}$ was used. The problem of the research is the selection of variables to the model C-D for the steel industry in Poland (on the basis of data for the period 2000-2015). The author of the article decided to verify the classical approach to define changed parameters of the C-D function according to the situation in Polish steel industry. The use of C-D function to assess economic activity of steel industry in Poland, evaluated the effectiveness of the restructuring measures (after the transformation of the economic system in Poland) and answer the question about level of the market maturity by restructured steel industry in Poland.

Keywords: Crude steel production, Polish steel industry, Cobb-Douglas production function

1. INTRODUCTION

The Cobb-Douglas production function, studied by econometrics, expresses the effect on the output of production factors: capital and labour [1]. The model can be used to describe the process of production that can be considered at the enterprise, branch and economy level [2]. Since the moment when C.W. Cobb and P.H. Douglas (1928) introduced the original definition of production function, many modifications were developed by their followers. These modifications were related to the scope of empirical research. Examples of applications can be found in the work of foreign researchers H.B. Chenery and P. Clark [3], A.A. Walters [4], M. Brown [5], M.D. Intriligator [6] or Polish: Z. Pawłowski [7], W. Welfe [8]. Extended versions of production functions (with certain assumptions) include function about constant elasticity of substitution (CES) [9]. In this publication, the Cobb-Douglas production function was used to develop a model on the industry scale, based on the restructuring of the steel industry in Poland. By modifying the production function, it has been claimed that this is Cobb-Douglas power function, which means that it is treated as a certain approximation of the actual form of production function of the authors. In the measurements of metallurgical production in Poland statistical data were used for 2000-2015, corresponding to the structure of the model: P (in the article record as Y) - dependent variable - steel production (natural units: tons), net production (pure) that is the added value of the steel industry, sold production of the steel industry (monetary units), L (in the article record as $X2$) working time (hours) - actual working time in the steel industry, labour costs: remuneration and benefits to employees in the steel industry (cash units), C (in article as $X1$) - explanatory variable - capital as the carrying amount of non-current assets (cash units) after depreciation and the degree of utilization of production potential by the steel industry in Poland. In terms of value, all units were recalculated by the GDP delta for 2000-2015, assuming the price from year 2015 as 100 %, thus yielding values at constant prices rather than current, which facilitated model comparison. The purpose of the study is to develop different models of production function models and to perform analysis of the elasticity of final production with respect to factors of production and the effects of production scale (production scale flexibility).

Hypothesis of the analysis: the selection of parameters (*L* and *C*) for the production function for the steel industry in Poland (according to the Cobb-Douglas production function) is the subject of econometric testing rules (as every econometric model) and the resulting model structure may differ from the baseline developed by Cobb and Douglas. The content of this publication is the presentation of structure of supporting production functions.

2. RESEARCH METHODS FOR USING MODELS OF C-D PRODUCTION FUNCTION MODELS

In the econometric analysis of C-D production functions, available and licensed software was included with the following auxiliary tools: EXCEL v. 2007 spreadsheet from MicroSoft, Statistical package Statistica v. 12 PL by StatSoft. In constructing econometric models, the classical least squares method was used to evaluate the linear parameter of formula 1 using the *Regression* tool from the Excel *Data Analysis* and the nonlinear estimation method: Gauss-Nevton and Levenberg-Marquardt numerical methods from Statistica.

$$\ln Y = a_0 + a_1 \cdot \ln X1 + a_2 \cdot \ln X2 \qquad (1)$$

where: *Y* - Production; *X*1 - Capital; *X*2 - Labour, *a*0, *a*1, *a*2 - model parameters.

Statistical verification of model fit was performed according to the following sequence of steps: 1. Estimation of model parameters; 2. Determination of determinant coefficient - R^2; 3.Test of *F* (Significance *F* for *p* less than 0.05, where *p* - significance level of the parameters: *X*1, *X*2); 4. Carrying of significance test of each explanatory variable (if *p* for each explanatory variable was less than 0.05, the explanatory variable had an effect on the explanatory variable). 5. Conducting the randomness test of the random component. The resulting power form of the production function is written in the form (formula 2).

$$\hat{Y}_t = a0 \cdot X1_t^{\,a1} \cdot X2_t^{\,a2} \qquad (2)$$

where: $\hat{Y}_t$ - production of the steel industry at the time (added value ie net production or sold production ie pure production), $X1_t$ - value of fixed assets in the metallurgical industry at the time (in total or less depreciation and utilization of production capacities by the steel industry in Poland); $X2_t$ - employees in the steel industry (persons) or actual working time in the steel industry (hours) or personnel costs (employee salaries and employee benefits) at the time; *t* - time, t=1.2...16 (appropriate for the scope of the study, which covered the years 2000-2015).

The research purposes (for empirical) abandoned the assumption that *a*1 + *a*2 > 0. After verifying models in econometric terms, they were also evaluated from the point of view of economics (productivity).

3. SELECTION OF VARIABLES FOR C-D EXTRUSION MODEL FOR THE STEEL INDUSTRY IN POLAND

3.1. Generally about restructuring changes in the steel production in Poland

During the transformation of the economic system in Poland (after 1989), the steel industry, as one of many industries, was a subject to restructuring processes in the 90s. The changes concerned not only steel production volume (quantitative reduction), but also qualitative changes resulting from realized investments and the impact of broadly understood technical and organizational progress. The steel industry withdrew (in 2002) the martensitic steel smelting technology. Before restructuring of the steel industry in Poland, more than 10 million tons of steel were produced, after production fell to around 9 million tones [10-11].According to the used technologies the structure of production has changed (increasing or decreasing the share of steel produced in electric furnaces or the converter steel). Until 2009, more steel was produced in the converters than in the electric furnaces. In 2009 several more electric processes were used to make steel. Between 2010-

2013, the proportions were the same (50 out of 50). After 2013 more amount of steel was produced in the converter, for example, in 2016 5.1 million tons of steel in converter (57 %) and 3.9 million tons in electric process (43 %) were produced in Poland [12]. In terms of employment, the number of employees was drastically reduced, and the structure of the workforce was also changed. In 1990, 147,000 people were employed in the steel industry, now more than 20,000 [13]. The structure of employees in individual positions also changes. According to the division of posts into workers and non-workers, despite the dominance of the former (the workforce in the steel industry accounts for about 75 % of the total posts), there has been an increase in the number of service and administration staff in recent years. Regarding the use of working time, there has been a decrease in time losses. The standard of working time in Poland also has changed (currently working less than 15 years ago). Long-term wage costs has increased, including employee benefits. The average wage in the steel industry is now over 6000 PLN, 15 years ago it was 3000 PLN.

3.2. Description of variables in the Cobb-Douglas production function in the analysis of Polish steel industry

The classical model of production function: *Y* - steel production (tons), *X*1 - fixed capital (money units) *X*2 - number of employees (persons) in power form, was not statistically correct (negative statistical verification due to used tests). In theoretical models (**Table 2**) $\hat{Y}$ was used for production. A further model was described which explained the added value generated by the metallurgical industry and was a net production gauge at constant deflator prices (this variable was found to be the basis for statistical modelling of Cobb - Douglas production function for the steel industry). In Poland in the years 2000-2015). Comparative models also used the production of the sold metallurgical industry in the years 2000-2015, also in fixed prices according to GDP deflator. In contrast to the factors of production, various degrees of disagreement have been introduced. Thus: the value of fixed assets in individual years was reduced by the depreciation costs. The data on the level of production capacity utilisation in the steel industry in Poland (**Table 1**) were also used in relation to the value of non-current assets. In terms of employment included the number of employees in total and broken down by job, the use of working time and labour costs (basic variables **- Figures 1- 4**).

Table 1 Steel production and capacity utilisation

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
million tons	10.50	8.81	8.37	9.11	10.58	8.33	9.99	10.63	9.73	7.13	7.99	8.78	8.34	7.95	8.56	9.20
%	83	70	66	72	84	66	79	84	77	57	63	70	66	63	69	73

Source: [12]

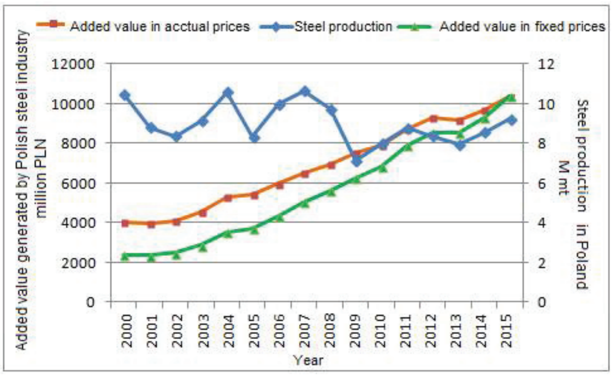

Figure 1 Steel production and added value generated by Polish steel industry [12]

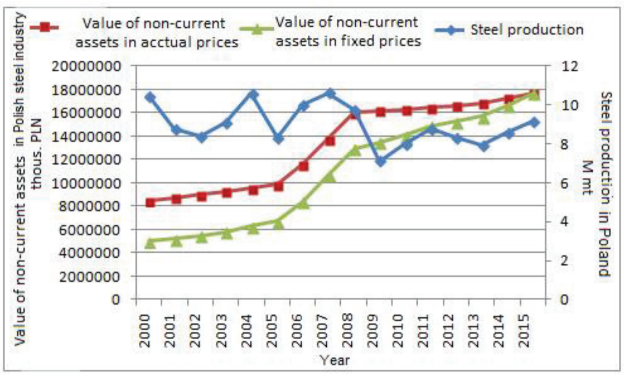

Figure 2 Steel production and value of non-current assets in Polish steel industry [12]

Maximum production capacity of 12.6 million tons (annual demand of the domestic economy for steel products is more than 12 million tons, apparent steel consumption in 2015 is 12.5369 million tons) [12]. The models presented in this publication are summarized in **Table 2**, which are discussed in accordance with the order number proposed in this table. The scope of interpretation included the coefficient of elasticity of production with respect to individual factors of production: $E_{\hat{Y}/X1}$ or $E_{\hat{Y}/X2}$ (formula 3) and the effect of the scale of production as the sum of the elasticity of the power function - *ESP* (formula 4); [14].

$$E_{\hat{Y}/X1} = \frac{\Delta \hat{Y}}{\hat{y}} \approx \frac{\Delta X1}{X1} \cdot a1 \;\; ; \;\; E_{\hat{Y}/X2} = \frac{\Delta \hat{Y}}{\hat{y}} \approx \frac{\Delta X2}{X2_2} \cdot a2 \quad (3)$$

$$ESP = E_{\hat{Y}/X1} + E_{\hat{Y}/X2} \quad (4)$$

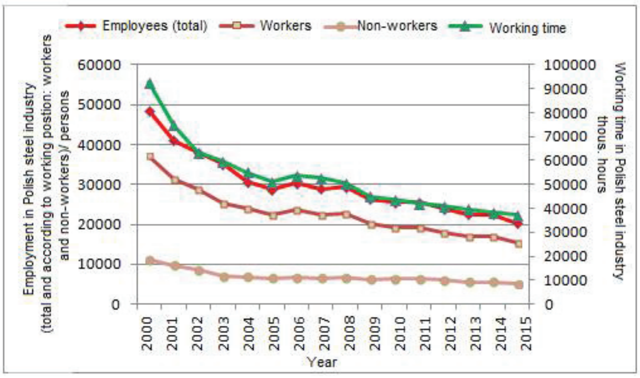

Figure 3 Number of employees (in total and broken down by job) and working time in Polish steel industry [12]

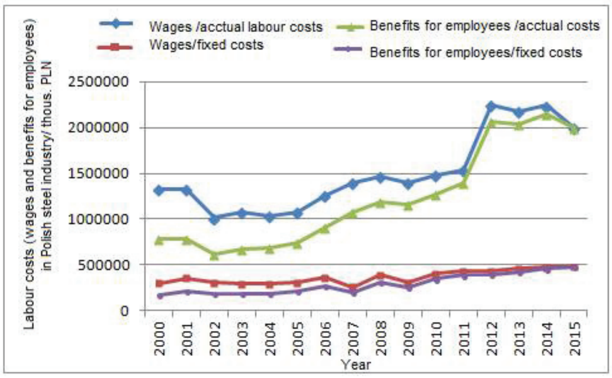

Figure 4 Labour costs in Polish steel industry [12]

In the first model (no. 1), the flexibility of production versus fixed assets reported that the increase of this property by 1 % resulted in an average increase in the value added of the steel industry in Poland by 0.7799 %, assuming that the level of employment would not change. On the other hand, the increase in the number of employees in the steel industry by 1 % causes an average decrease of 0.6371 %. The effect of the scale of production (elasticity of production) for this function 0.11276 indicated that inputs in the analysed function are growing at a faster rate than the effects, and since *ESP* < 1 is a case of decreasing production efficiency. For this function, the added value generated by the steel industry in Poland is growing at a slower rate than the total expenditure on increasing fixed assets and increasing employment. Disaggregating the number of employees to the level employed in production (workers employed in the ironworks in Poland) results in an increase in the effect of the production scale (*ESP* = 0.1428) - model no. 2. When labour expenditures were described by working time in the steel industry in Poland, a significant increase in the scale effect was obtained: *ESP* = 0.3605 (no. 3). In the models discussed so far, capital expenditures were described using fixed assets (annual values in 2000-2015). The coefficient of elasticity of production over fixed assets for the first two functions was the same ($E_{\hat{Y}/X1}$ = 0.7799), and in the third mode $E_{\hat{Y}/x1}$ = 0.8435. Disintegration, primarily through the depreciation level (annual average depreciation costs in fixed prices), and secondly by the capacity utilization rate in the steel industry in Poland (**Table 1**), resulted in a decrease in the production flexibility index relative to the detailed fixed assets compared to the no.1 model, respectively 0.0292% ($E_{\hat{Y}/X1}$ = 0.7507) and 0.1465% ($E_{\hat{Y}/X1}$ = 0.6334). On a comparable level was the rate of production elasticity described by the sold production of the steel industry at constant prices (no. 6 and 7): $E_{\hat{Y}/X1}$ = 0.6952, $E_{\hat{Y}/X1}$ = 0.7196. Flexibility ratios of $E_{\hat{Y}/X2}$ work productivity were negative (no. 1 to 7), the result of the negative correlation of variables *Y* and *X*2. It should also be emphasized that the effects of the scale of production on the production function as value added produced in the metallurgical industry were positive, then in the case of sold production those values were negative. Thus, the increase in the factor of production (in these models), ie the work measured by the number of employees or the working time by 1 %, caused the output of the factor function of the variable X2 to decrease accordingly. In models 1 to 7, scale effects (production scale flexibility) were less than 1, indicating a decrease in the productivity of the production factors. Beginning with model 8, the scale effects are greater than 1, and therefore there is increasing production efficiency (effects grow at a faster pace than total production). Models in which variable *X*2 is labour costs as annual employee salaries in fixed prices or as employee salaries, along with employee benefits, also at constant prices on a yearly basis, production flexibility (added value) Fixed assets, after depreciation and the utilization of production capacity in particular years in the steel industry, were respectively: $E_{\hat{Y}/X1}$ = 0.6843 $E_{\hat{Y}/X1}$ = 0.7150 (condition *X*2 is unchanged), and production rates in relation to labour costs $E_{\hat{Y}/X2}$ = 0.4840, $E_{\hat{Y}/X2}$ = 0.4284 (when *X*1 unchanged).

Table 2 A summary of Cobb - Douglas production function used for the steel industry in Poland

No. models	Form of model	Parameters	$E_{\hat{Y}/X1}$	$E_{\hat{Y}/X2}$	*ESP*
1.	$\hat{Y} = 14308.22 \cdot X1^{0.7799} \cdot X2^{-0.6523}$	$\hat{Y}$ - added value *X*1 - value of fixed assets *X*2 - number of working people	0.7799	-0.6523	0.1276
2.	$\hat{Y} = 10273.10 \cdot X1^{0.7799} \cdot X2^{-0.6371}$	$\hat{Y}$ - added value *X*1 - value of fixed assets *X*2 - number of working people in working positions	0.7799	-0.6371	0.1428
3.	$\hat{Y} = 1180.25 \cdot X1^{0.8435} \cdot X2^{-0.4830}$	$\hat{Y}$ - added value *X*1 - value of fixed assets *X*2 - actual working time	0.8435	-0.4830	0.3605
4.	$\hat{Y} = 24929.4 \cdot X1^{0.7507} \cdot X2^{-0.6558}$	$\hat{Y}$ - added value *X*1 - value of fixed assets less depreciation costs *X*2 - number of working people	0.7507	-0.6558	0.0949
5.	$\hat{Y} = 4527532 \cdot X1^{0.6334} \cdot X2^{-0.9841}$	$\hat{Y}$ - added value *X*1 - value of fixed assets less depreciation costs and taking into account the capacity utilization rate *X*2 - number of working people in working positions	0.6334	-0.9841	-0.3507
6.	$\hat{Y} = 8455717.1 \cdot X1^{0.6952} \cdot X2^{-1.0313}$	$\hat{Y}$ - sold production *X*1 - value of fixed assets *X*2 - number of working people	0.6952	-1.0313	-0.3361
7.	$\hat{Y} = 3089785.9 \cdot X1^{0,7196} \cdot X2^{-0,9214}$	$\hat{Y}$ - sold production *X*1 - value of fixed assets *X*2 - actual working time	0.7196	-0.9214	-0.2018
8.	$\hat{Y} = 0.1148 \cdot X1^{0.6843} \cdot X2^{0.4840}$	$\hat{Y}$ - added value *X*1 - value of fixed assets less depreciation costs and taking into account the capacity utilization rate *X*2 - labour costs (wages and benefits for employees)	0.6843	0.4840	1.1683
9.	$\hat{Y} = 0.1715 \cdot X1^{0.7150} \cdot X2^{0.4284}$	$\hat{Y}$ - added value *X*1 - value of fixed assets less depreciation costs and taking into account the capacity utilization rate *X*2 - labour costs (wages for employees /employee salaries)	0.7150	0.4284	1.1434

Grey colour for function graphs are shown on **Figure 5**.

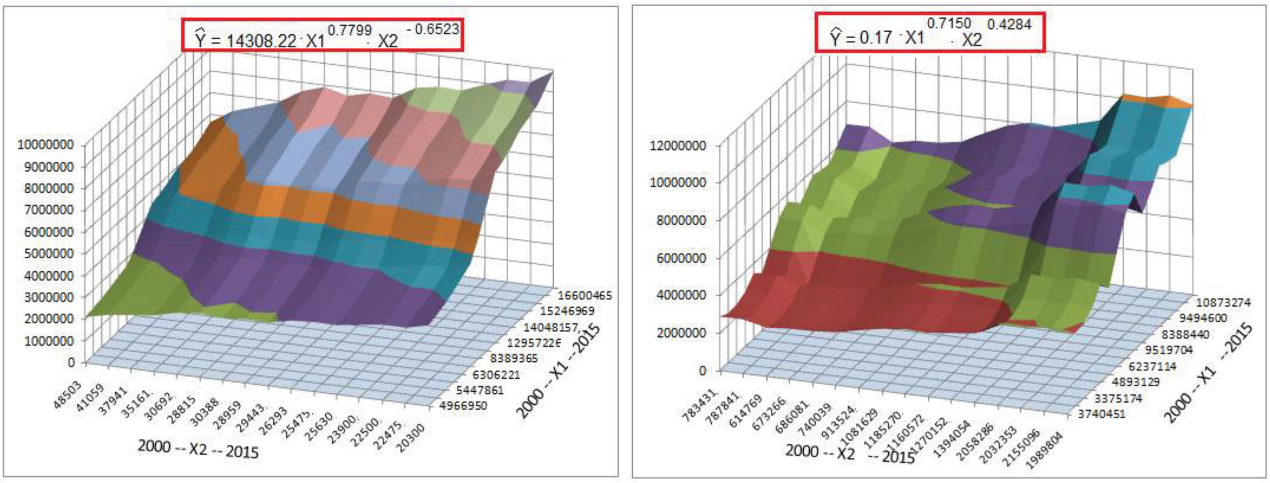

Figure 5 Function graphs of Cobb-Douglas production function - models for Polish steel industry

Source: Own research

Models with indexes: In the last three models (no. 10, 11, 12 in **Table 3**), specific performance indicators were used to describe workloads:

Index no.1: Occupancy rate as the ratio of the number of employees in the non-staff to the number of employees in the workforce and the index of actual working time according to positions as the ratio of the actual working time in the non-working positions to the actual working time in the workplace.

Index no. 2: Efficiency ratio of working time, which is the quotient of the total time spent by the time the maximum (possible to be worked in the industry, taking into account the loss of time due to workers' rights, eg. the right to leave).

Index no. 3: The ratio of the effective working positions as the product of the ratio of the effective operation time of the white-collar worker and the relative positions of the white-collar workers.

Table 3 C-D models for the steel industry in Poland with specific performance indicators

No. models	Form of model	Parameters	No. indexes	$E_{\hat{Y}/X1}$	$E_{\hat{Y}/X1}$	ESP
10.	$\hat{Y} = 0.4399 \cdot X1^{0.7434} \cdot X2^{0.3587}$	*Y* - added value *X*1 - value of fixed assets less depreciation costs and taking into account the capacity utilization rate *X*2 - labour costs (employee salaries) after the proper index	1	0.7434	0.3587	1.1021
11.	$\hat{Y} = 0.1973 \cdot X1^{0.7054} \cdot X2^{0.4311}$		2	0.7054	0.4311	1.1365
12.	$\hat{Y} = 0.4715 \cdot X1^{0.7367} \cdot X2^{0.3609}$		3	0.7367	0.3609	1.0976

Source: Own research.

4. CONCLUSION

Models are used to describe many aspects of the functioning of metallurgical enterprises, including process management [15] and technology [16]. In this work, the scope of the production function has been narrowed. Evaluation of the effectiveness of the restructuring process in the steel industry in Poland is possible in various research areas. One of them is the analysis of C-D production function. On the basis of models have been found:

1) Negative correlation between variable *Y* (output): added value or sold production and variable *X*2 (labour input - *L*); number of employees or actual working time in the steel industry in Poland in 2000-2015.

2) Positive correlation between variable *Y* (output): added value or sold production and variable *X*1 (capital outlay - *C*): total fixed assets or depreciation costs and capacity utilization in the steel industry in Poland.
3) Most similar models of production functions for models of authors C.W. Cobb and P.H. Douglas received in the case of labour cost description (employee and employee benefits paid in 2000-2015) and in the case of the use of labour conversion formulas by appropriate performance indicators.

Reference to the hypothesis of the analysis: based on the restructuring changes in the steel industry in Poland, production functions with changed parameters were obtained. Modifications were made to parameter C, that is, capital which was realized by the degree of its use for production and the deprecation and to parameter L: time work and labor costs.

REFERENCES

[1] COBB, C. W., DOUGLAS, P. H. *A* Theory of Production*, American Economic Review*, 1928, vol. 8, no. 1, pp.139 - 165.

[2] SAMUELSON ,P. A. Paul Douglas's Measurement of Production Functions and Marginal Productivities, *Journal of Political Economy*, 1979, vol. 87, no. 5, pp. 923 - 939.

[3] CHENERY, H.B. and CLARK P. *Interindustry economics*. New York: John Willey & Son, Inc. 1959.

[4] WALTERS, A.A. Production and cost functions: An econometric survey*, Econometrica, 1963, no.31, pp.1-66.*

[5] BROWN, M., ed. The theory and empirical analysis of production, *Studies in Income and Wealth*, vol. 31, National Bureau and Economic Research. New York: Columbia University Press, 1967.

[6] INTRILIGATOR, M.D. Mathematical optimization and economic theory, New Jersey: Prentice Hall, Englewood Cliffs, 1971, 501p.

[7] PAWŁOWSKI, Z. An econometric analysis of the production process. Warsaw: PWN, 1976, 242p.

[8] WELFE, W. The econometric models of the Polish economy. Warsaw: PWE, 1992, 358p.

[9] ARROW, K.J, CHENERY, H., MINHAS, B. And SOLOW R.W. Capital-labor substitution and economic efficiency, *The Review of Economics and Statistics*, 1963, vol. 42, pp.225-50.

[10] GAJDZIK, B. Prognostic modelling of total steel production and according to production technology in Poland, *Metalurgija*, 2017, vol.56, no. 1-2, pp. 241- 244.

[11] GAJDZIK, B. A*nalysis of the size of steel production in Polish steel industry* [in] 25th Anniversary International Conference on Metallurgy and Materials, May 25ty-27th 2016, Brno.

[12] Polish steel industry, reports of Polish Steel Association (2000-2016), Katowice: Polish Steel Association; www.hiph.org.

[13] GAJDZIK, B., SZYMSZAL J. Generation Gap management in restructured metallurgical enterprises in Poland, *International Journal of Management and Economics*, 2015, no. 47, pp. 107-120; http://www.sgh.waw.pl/ijme/.

[14] WELFE W. WELFE A., *Ekonometria stosowana*, PWE, Warszawa 2004.

[15] ZWOLIŃSKA, B. Matematyczny model analizy strumienia jakości - Model of the analysis of the quality stream *Autobusy : technika, eksploatacja, systemy transportowe* ; 2016, no.12, pp. 1906-1913.

[16] SZCZUCKA-LASOTA, B.; WEGRZYN, T., STANIK, Z.; et al. Selected parameters of micro-jet cooling gases in hybrid spraying process, *Archives of Metallurgy and Materials*, 2016, vol. 61 Issue: 2, pp: 621-624.