VIEW FACTORS FOR THE CRUCIBLE FURNACE

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Abstract

The crucible furnaces are mainly used for melting of metals or for keeping the metal in the liquid state. Among other things, an applicable mathematical optimization of the crucible parameters could contribute to reducing the high energy intensity of crucible furnaces. The component part of the considered mathematical model of the radiation heat transfer is the calculation of the view factors whose analytical solution is possible only in special cases. This article deals with the proposal to simplify the analytic calculation of the local view factors for the generally shaped crucible on standard conditions.

Keywords: *Crucible furnace*, view factor

1. INTRODUCTION

The energy intensity of the crucible furnaces that are mainly used in the metallurgy for melting as well as for keeping the melt in the liquid state is very high. The energy consumption can be together with many others factors influenced by the shape of the crucible. Some sophisticated and expensive experiments used for evaluating of various crucible shapes or for new shapes designing can be replaced by the mathematical model application. The mathematical models in which the numerical calculations of various relations and dependencies are replaced by analytic expressions of these seem to be generally more applicable in the research.

The local view factor that is used for the radiation heat transfer calculation cannot be analytically stated in general form. In our previous papers compiled in [1] we presented the analytic, but only approximate calculation of the local view factor for some crucible shapes. In this article we considerably extend the simplified analytic calculation of the local view factor for the general crucible shape defined in the following.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The calculation of the radiation heat transfer between the crucible (the solid A bounded by the surface S_A) and the furnace (the solid B with the interior surface S_B) can be mathematically expressed by the surface integral over the surface S_A whose integrand is a surface integral over the surface S_B [2].

The base of the calculation over the surface S_B is the so-called *local view factor*. The local view factor (only the view factor in the following) for the radiation from the surface element dS_A to a certain region of the surface S_B is given by the relation

$$\Phi_{A,B} = \frac{1}{\pi} \int_{S_{B*}} \frac{\cos\alpha_A \cdot \cos\alpha_B}{d^2} \cdot dS_B \ , \tag{1}$$

where dS_B is the surface element of the surface S_B. The angles α_A, α_B are the angles between the normal lines of elements dS_A, dS_B and the line connecting both elements; d is the distance of the elements. The furnace interior surface is the cylindrical surface that is for practical calculations divided into specific subareas (e.g. the bottom, the top and the interior wall). The above mentioned integral is not calculated over the whole surface S_B but only over its ‘visible’ part denoted S_{B^*}. The element dS_A namely radiates only to the corresponding exterior half-space.

The analytic calculation of the view factor according (1) is possible only in special cases (e.g. [3]). The following largely applicable concept of approximate analytic expression of the view factor is related to the following conditions (that are commonly fulfilled in practice):

- The crucible is the solid of revolution with the vertical axis.
- The interior surface of the furnace has a shape of the right circular cylinder with the vertical axis.
- The crucible is centrally placed in the cylindrical furnace, i.e. the surfaces S_A and S_B are rotational and co-axial.
- All points of the surface S_A in the same height have the same temperature and also the temperature of all points of the surface S_B depends only on their height (measured from the bottom of the crucible).

For the mathematical description of the problem we use the cylindrical coordinate system determined by the axis and the bottom of the furnace (the bottom coordinate is equal to zero). The coordinates are denoted as:

- r - radius,
- h - height.

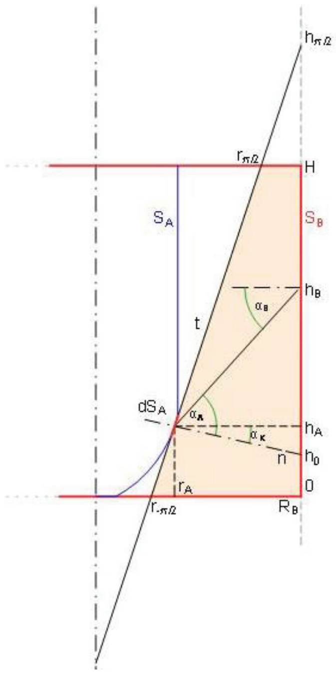

Figure 1 A convex crucible

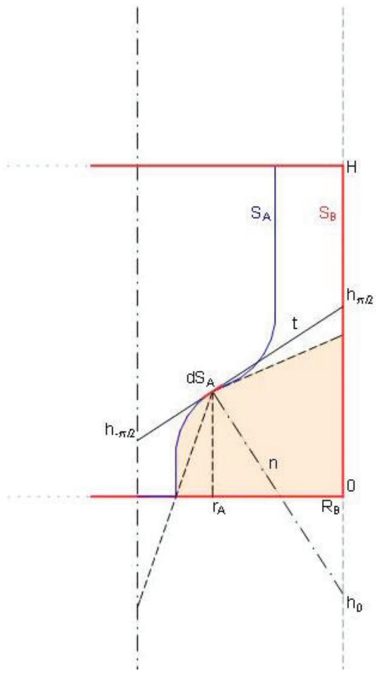

Figure 2 A nonconvex crucible

Moreover, the following nomenclature and designation will be applied (see **Figures 1, 2**):

- R_B - (interior) radius of the furnace,
- H - (interior) height of the furnace,
- r_A, h_A - coordinates of the element dS_A (radius, height),
- α_K - angle between the normal line of the element dS_A and the horizontal plane,
- t , n - tangential plane and normal line of the crucible surface in the ‘point’ dS_A,
- h_0 - height in which the normal line n intersects the extended sector of the element dS_A (the sector is defined in the next section),
- $h_{\pi/2}$ - height in which the plane t intersects the extended sector of the element dS_A ,

- $h_{-\pi/2}$ - height in which the plane *t* intersects the furnace axis.

If $\alpha_K = 0$ then $h_{\pi/2} = +\infty$, $h_{-\pi/2} = +\infty$.

In a view of mentioned assumptions, for calculation of the radiation heat transfer we find sufficient to know the view factors of the element dS_A related only to the following ***partial surfaces***:

- the interior wall of the furnace or this wall belt between the heights h_1 and h_2,
- the bottom and the top of the furnace or their annulus determined by the radii r_1 and r_2.

The general crucible shape mentioned in the introduction section results from current shapes of crucibles. In this article we consider ***the general crucible shape*** as the shape which section (i.e. the vertical cut intersecting the crucible axis) can be described by arbitrary non-decreasing function $r = f(h)$. Thus, the radius increases or remains identical with the increasing height, i.e. the crucible does not necessarily have to be a convex solid. The crucible can radiate to itself but its section is not allowed to contain inflexions (because in such case more than one radius *r* would be equal to one height *h*). Contrary to the case of a nonconvex crucible the element dS_A of a convex crucible can radiate only to the interior wall, the top and the bottom of the furnace (not into any part of the crucible). **Figure 1** illustrates some crucible with the convex section - the *convex crucible* - and **Figure 2** illustrates some *nonconvex crucible*.

We find useful to note that, for plasticity, all illustrated crucibles have less real shape (namely **Figure 2**) and their maximal diameters are significantly less than the interior diameter of the furnace.

3. ADVANCED CALCULATION OF THE LOCAL VIEW FACTOR

Advanced simplification of view factor calculation results from the fact that the surfaces S_A a S_B are rotational and co-axial. **Figure 3** illustrates the horizontal cut of the surfaces S_A and S_B where the solid red line represents two elements dS_{A1} and dS_{A2} of the same size and placed in the same height. Each of these elements determines elementary cylindrical sector and the corresponding surface of the same size on the furnace interior wall which we refer as the ***sector*** in the following. In **Figure 3** the sectors are represented by thick black arcs. **Figure 3** can also contribute to clarification of the ***source theorem***: The quantity of the radiation that the element dS_{A1} radiates to the fixed height of the sector of the element dS_{A2} is equal to the quantity of the radiation that the element dS_{A2} radiates to the same height of the sector of the element dS_{A1}.

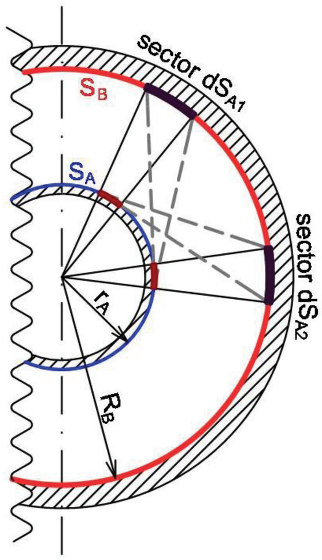

Figure 3 A scheme for the source theorem

The source theorem led us to the first ***simplifying assumption***: The distribution of the view factors of the element dS_A into mentioned partial surfaces on the crucible furnace surface is approximately equal to the distribution of the view factors of the element dS_A into the intersections of these partial surfaces to the sector of the element dS_A.

The resulting generalization of the source theorem for the infinitely high cylindrical surface which is the extension of the cylindrical surface of the furnace interior wall (**Figure 4**) allows us to calculate the view factors for the bottom of the furnace, for the top of the furnace or for annuluses of these. The calculation is realised by the projection of the view factor on the ***extended sector of the element dS_A*** or on the axis of the furnace. In **Figure 4** the extended sector of the element dS_A looks like the 'infinite'

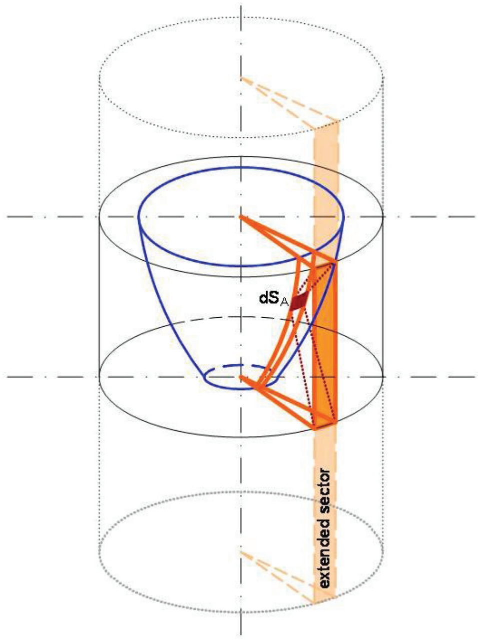

Figure 4 The sector and the extended sector of the element dS_A

strip (light-coloured) which part is the sector of the element dS_A (dark-coloured). In the mathematical point of view the extended sector of the element dS_A reduces to the line and the sector of the element reduces to a segment of this line.

Therefore the approximate analytic expression of the view factors was derived under the fictitious assumption that each element radiates only to its own extended sector or to the furnace axis. If any part of mentioned radiation is absorbed by the crucible or by the bottom or the top of the furnace then this part of radiation is used for calculation of the corresponding view factor.

The fictitious height h_i, corresponding to some fixed point on the bottom or the top of the furnace or on the surface of the crucible is obtained by the projection of this point from the element dS_A on the extended cylindrical surface of the furnace or on the furnace axis (examples of the projection can be found in [1]).

The advanced calculation of the required view factors is based on calculation of the integral $I(h_1,h_2)$ that represents the quantity of the radiation absorbed by the extended sector of the element dS_A between the heights h_1 and h_2:

$$I(h_1,h_2)=q\cdot\left(\cos\alpha_K\cdot\int_{h_1}^{h_2}\frac{(R_B-r_A)^2}{\left((R_B-r_A)^2+(h-h_A)^2\right)^2}\cdot dh-\sin\alpha_K\cdot\int_{h_1}^{h_2}\frac{(R_B-r_A)\cdot(h-h_A)}{\left((R_B-r_A)^2+(h-h_A)^2\right)^2}\cdot dh\right)\quad, \tag{2}$$

where

$$q=\frac{2(R_B-r_A)}{\pi\cos\alpha_K+2\sin\alpha_K}\quad. \tag{3}$$

In [1] the integral $I(h_1,h_2)$ was presented only for the vertical element dS_A , i.e. for $\alpha_K = 0$. The integral was standardized to be equal to 1 on the interval $(-\infty,+\infty)$ (according to the definition the sum of view factors of the element dS_A on all visible surfaces must be equal to 1). For arbitrary element dS_A (i.e. for $\alpha_K \in \langle 0,\pi/2)$) the standardization of the integral $I(h_1,h_2)$ is determined by the requirement $I(h_0,h_{\pi/2}) = ½$.

In the case of a convex crucible the integral $I(-\infty,h_{\pi/2})$ represents the sum of the view factors on the visible part of the furnace top and interior wall and on the visible part of the annulus of the furnace bottom for $r \geq r_A$. The value $1 - I(h_0,h_{\pi/2})$ then represents the view factor for the remaining visible part of the furnace bottom. Thus, the view factor for the whole visible part of the bottom is the sum $I(-\infty,h_0) + (1 - I(-\infty,h_{\pi/2}))$.

In the case of a nonconvex crucible of mentioned general shape the element dS_A can radiate to the part of the crucible that is situated above or below the element. The corresponding view factors then are contained in the value $I(-\infty,h_{\pi/2})$ or $1 - I(-\infty,h_{\pi/2})$.

We use the integral $I_o(h_1,h_2)$ to calculate view factors for such parts of the furnace bottom or of the crucible that are projected on the furnace axis. The integral $I_o(h_1,h_2)$ is similar to the integral $I(h_1,h_2)$ but the invariable q is estimated under the requirement $I_o(-\infty,h_{-\pi/2}) = 1-I(-\infty,h_{\pi/2})$.

4. ILLUSTRATION OF RESULTS

For the practical use of the presented approximate calculation of the local view factor we create new program code in Matlab. The program enables to enter arbitrary crucible shapes that fulfil the above mentioned conditions. The program also makes possible to calculate view factors for arbitrary subareas of the furnace interior surface. If the temperature of the furnace interior surface is constant we can calculate the view factor only for the bottom, the interior wall and the top of the furnace. To illustrate calculation results we chose three simple crucible shapes: cylindrical, conical and rounded (similar to the shape in **Figure 1**). All chosen shapes are obviously convex.

For better plasticity, the second and the third illustrated crucibles have again less real shape and also the ratio of the furnace height to its radius is less real.

In calculations the following data were used: the furnace radius 8 units, the furnace height 7 units, the maximal radius of the crucible in all chosen cases 6 units (other crucible sizes can be seen in **Figure 5**). The unit can represent 1 cm or 1 dm or 1 inch. The view factors depend only on the ratios of these sizes.

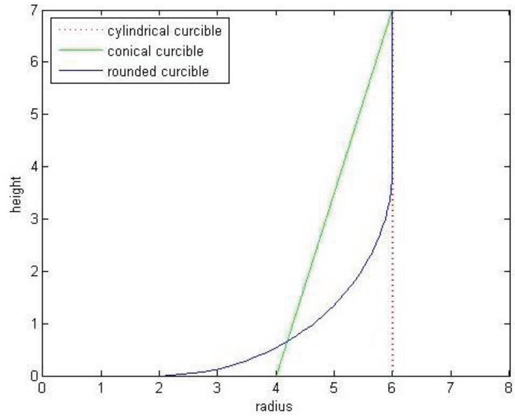

Figure 5 Profile curves of three chosen crucible shapes

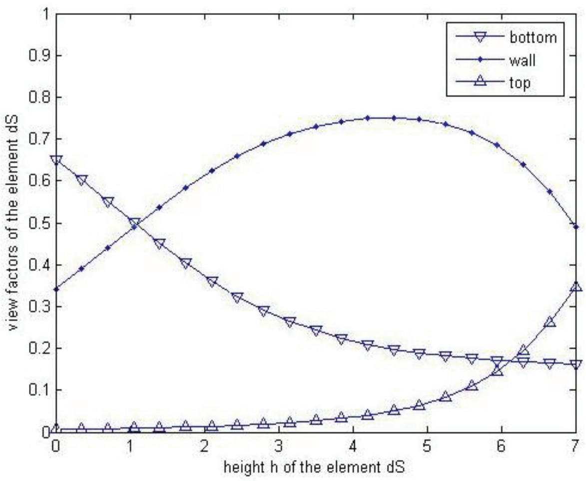

Figure 6 Graph for the conical crucible

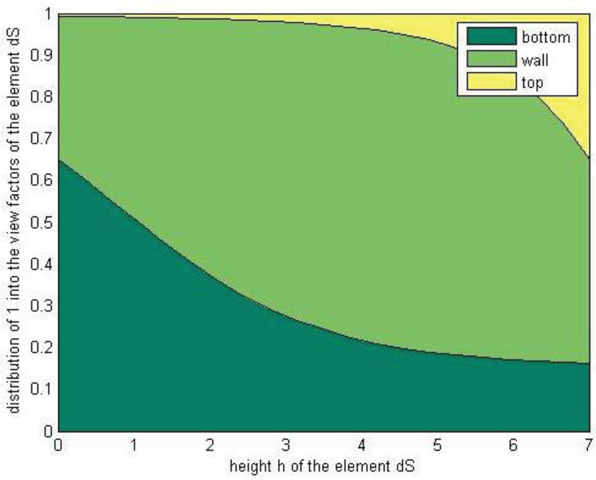

Figure 7 Graph for the conical crucible

The **Figure 6** illustrates the dependence of the view factors for the bottom, the interior wall and the top on the height *h* of the element dS_A, thus the figure holds three curvilinear graphs. In view of the fact that the crucible is convex, the sum of three values of the view factors appropriate for arbitrary height *h* of the element dS_A must be equal to 1. This fact is better reflected by the area graph (or the zone graph) in the **Figure 7**. In the area graph the value of the dependent variable (the view factor) is not generally represented by the vertical coordinate but by the height of the appropriate zone. In the **Figure 7** the horizontal axis also describes the height level of the element dS_A. In the vertical direction the magnitudes of view factors on the visible parts of the bottom, the interior wall and the top of the furnace are represented through the use of three colours. The **Figure 7** clearly illustrates how, in the dependence on the increasing height *h* of the element dS_A, the view factor for the bottom predominates at first. The view factor for the top is small at first and significantly increases only in the end.

In the **Figures 8** and **9** there are only the area graphs for the rounded crucible and for the cylindrical one. The area graphs in the **Figures 7, 8** and **9** obviously illustrates that the shape of the crucible considerably influences the distribution of observed view factors.

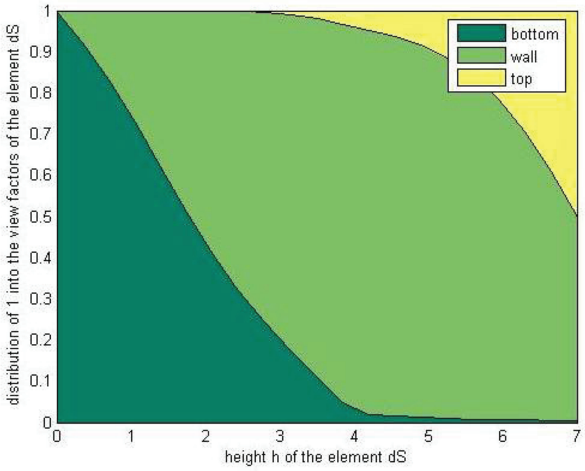

Figure 8 Graph for the rounded crucible

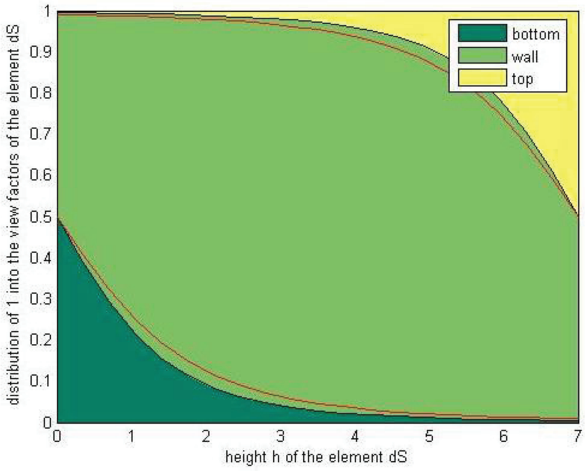

Figure 9 Graph for the cylindrical crucible

In view of the available information only the results obtained for the cylindrical crucible can be confronted with the reality. In the area graph in the **Figure 9** the red curves represents the boundaries between coloured areas obtained by the accurate calculation according to [2]. The area graph for the cylindrical crucible is symmetric with respect to its centre (the view factor for the bottom for the height *h* is equal to the view factor for the top for the height *H-h*).

5. CONCLUSION

The presented approximate calculation of the local view factor for the crucible of the mentioned general shape can be simply used for calculating of the radiation heat transfer not only in the case of constant temperature of the crucible furnace but even in the case when the temperature of the furnace interior wall depends on the distance from the furnace bottom and when the temperature of the bottom and top of the furnace depends on the distance from the furnace axis. The presented calculation also takes into account such cases when some part of the crucible radiates to another part of the crucible.

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