NON-PARAMETRIC ASSESSMENT OF THE UNCERTAINTY IN THE ANALYSIS OF THE AIRFOIL BLADE TRACES

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Abstract

The paper describes a non-parametric approach to the statistical analysis performed on data obtained during investigation on the airfoil blade traces. This approach releases us from limits of many conditions. Typical statistical analysis of data, with factorial (qualitative) control approach, bases on many theoretical assumptions, with the most important and the most influential one: the normality of random noises distributions (usually named 'errors'). The solution is the bootstrap approach, originally proposed by Efron in 1979, later in 1995 described in details by Shao and Tu. The bootstrap is formally the method based on random resampling with replacement from the source dataset. It allows to identify the whole distribution shape of raw data and related confidence intervals without additional assumptions about particular distribution. The paper presents the practical case: the factorial approach performed for the fixed-effect analysis of the relation between secondary dendrite arm spacing and carbides for the airfoil blade traces. The paper contains notes on encountered difficulties and possible guidelines for similar analysis.

Keywords: Uncertainty, non-parametric approach, SDAS, carbide, superalloy

1. INTRODUCTION

Experiments may be conducted and analyzed in two general manners [1, 2]:

- in the passive mode, when the experimenter is only an observer noting values (if observed variables are quantitative) or labels (if observed variables are qualitative) describing the state of the investigated object or process; in such experiment the cause-and-effect relationships may be unknown [3],
- in the active mode (usually known as designed experiment), when the experimenter actively changes settings of the investigated object or process e.g. [4], typically according to a previously prepared plan named experimental design; in such experiment the cause-and-effect relationship need to be known before the experiment is conducted [3, 5].

The industrial practice focuses only on the second kind of experiments i.e. on the designed experiments [6]. The designed experiments were known in 19th century however the process of their construction and analysis were not formalized. The first scientist who prepared designed experiments and their analysis based on strict mathematical formulas was R.A. Fisher. His concept, known as "latin squares" [7], was derived from the analysis of variance (ANOVA) procedure [8]. Later, his idea was developed into factorial designs by Yates [9], response surface methodology by Box [10] and finally formalized by Kiefer and Wolfowitz [11]. The formal mathematical model [12] founding the core of such domain known as 'design of experiments' includes the specific random term describing 'noise factors'. The noise factors are the surrogate name for all uncontrolled environmental disturbances. The well-known parametric analysis assumes that the joined effect of noise factors is described by the normal distribution with the mean of zero value and the unknown variance [6, 13]

i.e. N(0,σ). The rest of the analysis procedure bases on this assumption and leads to the *t* Student distribution as a shape describing probabilistic uncertainty of the obtained results [6, 13].

Practical observations, especially well-known box-plots, lead to the conclusion that such assumption is often weakly satisfied [14, 15] and as the result one cannot be sure about *t* Student distribution as a shape of results uncertainty distribution and bounds of confidence intervals [16-19]. The solution of such problem are analytical procedures weakly related to the specific probabilistic distribution. They are known under common name 'non-parametric methods'. This set of methods includes among others solutions based on ranks e.g. Kruskal-Wallis ANOVA [20], solutions based on re-sampling e.g. Efron's bootstrap [21] or the specific solution based on Wilkes's theorem i.e. Owen's empirical likelihood ratio [22, 23]. However both solutions, Efron's and Owen's base on an intensive numerical computations but Efron's solution is easy to implementation while Owen's rather not. The non-parametric methods are especially useful for the fuzzy approach [24, 25] which is very suitable for vague data, uncertain data or incomplete datasets [26, 27].

The following sections will discuss and apply the bootstrap method to analyze the influence of the trace location onto the SDAS measurement.

2. MATERIALS

Nickel-based superalloys are used mainly in turbines and produced in an investment casting process useful to obtain complex shape geometry. Studies were performed on the IN 713C superalloy [28]. Castings were cut off and the cross-sections were prepared as metallographic samples with the etched surface. The microstructural investigations of the cross-sections were conducted on Hitachi S-4200 SEM (**Figure 1**).

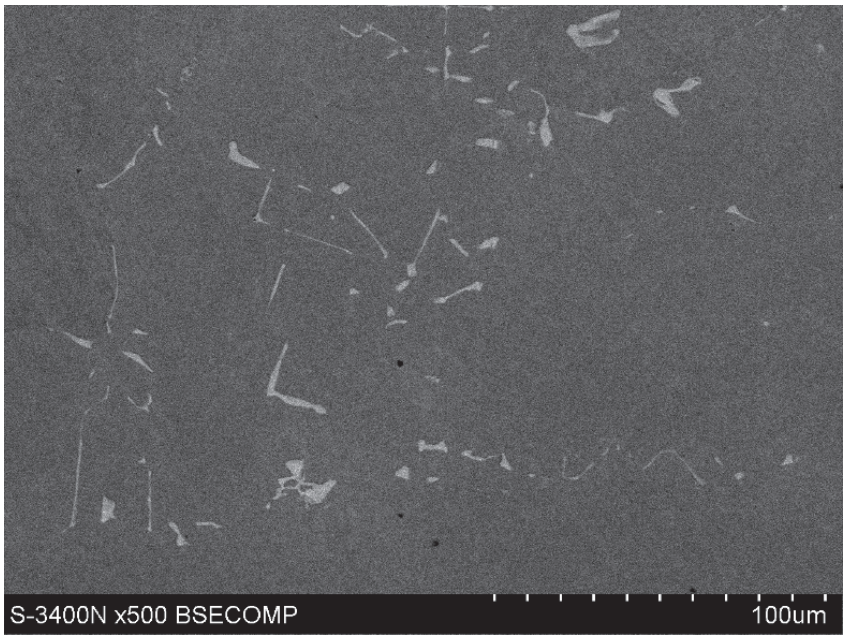

Figure 1 The carbides detected in the selected image of a cross-section

Next, microphotographs were analyzed by image analysis methods [29] (Met-Ilo program) to estimate quantitatively parameters describing (γ + γ') eutectic islands occurred in the superalloy. Six different traces were analyzed resulting in the dataset of 6 groups [30] with 31, 49, 80, 75, 64 and 61 values, respectively. The dataset contained secondary dendrite arm spacing (SDAS). The obtained raw data were compared to find if the different location of traces differentiate the measurement of SDAS.

3. METHODS

3.1. Boostrap approach

The bootstrap method, described in details by Shao and Tu [31], bases on processing of the dataset iteratively re-sampled from the original raw dataset. The idea of the method is presented in **Figure 2**.

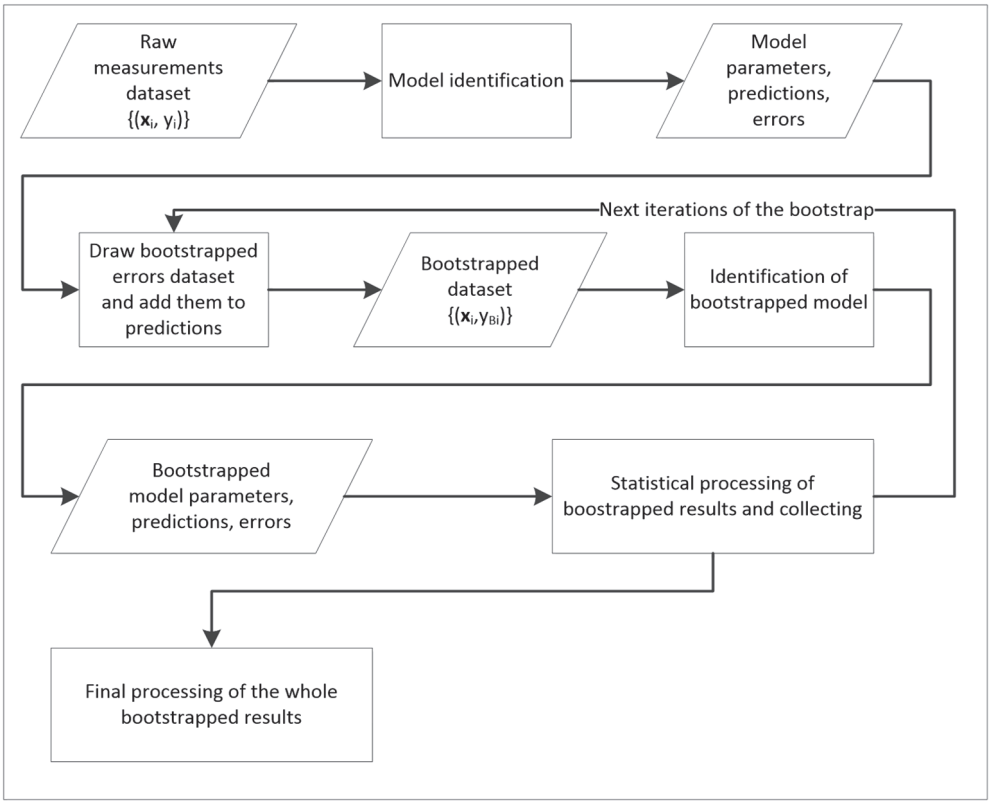

Figure 2 The scheme of the bootstrap method based on residuals

The key issue in the bootstrap is to make a proper identification of the random term built into the analyzed process. The main assumption of the bootstrap is focused on this term: its realizations should be independent and identically distributed (acronym i.i.d). In the passive experiment, the realization of the term is a whole measurement. In the active experiment, where a model is built according to the assumed cause-and-effect relationship, usually an error of the model (difference between prediction and measurement) is treated as a realization of i.i.d term.

It leads to the following algorithm: process a raw dataset, identify parameter of an assumed model, evaluates its errors and next iteratively randomly draw new errors dataset from the original errors dataset, add them to original predictions generating bootstrapped predictions, identify the bootstrapped model and process it further collecting obtained results. After a large number of iterations, collected results may be evaluated by statistical methods resulting in a whole distribution instead of the one number as in classic approach.

3.2. Predictive model

The predictive model was selected as a fixed-effects model with one factor. The model is constructed from some additive terms: average response μ and vector of main (linear) effect dependent on the single factor (trace id).

The model has the following formula:

$$y_i = \mu + A_i \tag{1}$$

where:

y_i - measured eutectic area for *i*-th trace,

μ - average measurement,

A_i - main (linear) effect of trace id,

i - enumerator of a investigated traces (6 pieces).

4. RESULTS AND DISCUSSION

The analysis of the raw dataset led to the following base model:

$$y = 26.13 + \mathbf{A}\begin{bmatrix} 8.32_{T1} \\ -4.18_{T2} \\ -2.15_{T3} \\ 0.72_{T4} \\ -1.04_{T5} \\ -1.67_{T6} \end{bmatrix} \tag{2}$$

Its main effect plot is presented in **Figure 3**.

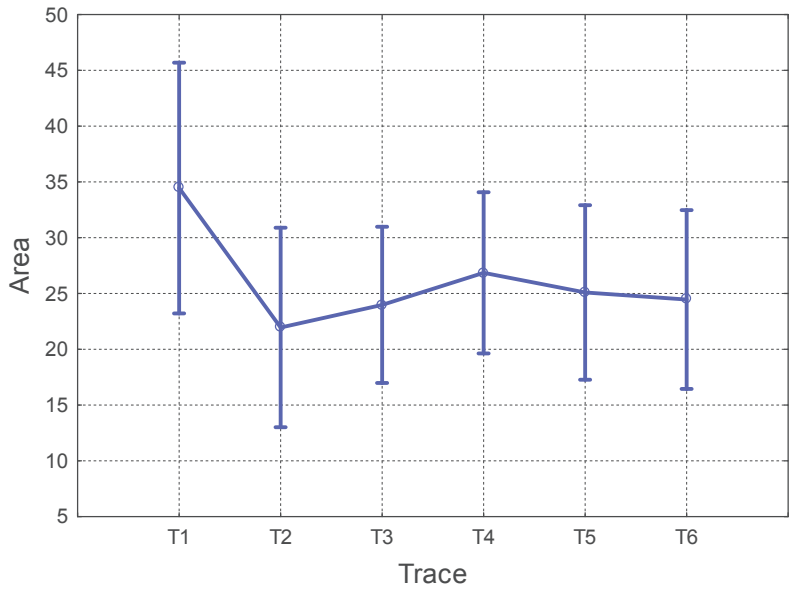

Figure 3 The main effects plot revealing the strong influence of a trace location on measured SDAS

The bootstrap was performed for 10000 iterations, because such a number allows easy identification of bounds for 95 % confidence interval (located at 250 and 9750 positions inside sorted dataset columns). The yielded statistics for the model constant and effects are presented in **Table 1** simultaneously with values obtained from classic parametric analysis. As one can see, the bootstrap gives values unavailable in classic approach i.e. bounds of the confidence interval for T6 effect.

Tests of the normality were rejected for all effects T1…T6. Only constant was not rejected however at very low p-Value = 0.06. It reveal that classic assumption of normality is not satisfied and bootstrap based results and more reliable in this context. The confidence interval bounds for all effects have different signs i.e. zero value is located inside confidence intervals. It may be interpreted as the statistical non-significance for all effects. It means that location of traces has no significance influence on measured values of SDAS.

Table 1 Descriptive statistics of the model constant and effects obtained from the bootstrap simulation

Model term	Mean		-95 % CI		+95 % CI	
	classic	bootstrap	classic	bootstrap	classic	bootstrap
constant	26.13	26.14	22.67	22.82	29.60	29.73
T1	8.32	8.30	-1.49	-0.81	18.13	18.64
T2	-4.18	-4.14	-12.26	-11.55	3.90	4.26
T3	-2.15	-2.18	-8.83	-8.55	4.53	4.65
T4	0.72	0.75	-6.12	-5.80	7.56	7.73
T5	-1.04	-1.00	-8.30	-7.93	6.23	6.69
T6	-1.67	-1.72	-	-5.88	-	8.87

5. CONCLUSIONS

1) The investigation revealed that the location of traces has no significance influence on measured values of SDAS.
2) The bootstrap approach revealed that the normality assumption assumed in the classic approach is not satisfied in this data and cannot be set as a base for reliable statistical inference.
3) The yielded values obtained from the bootstrap approach are consistent with means obtained from the classic approach however bounds of the confidence intervals differ even up to 46 %.
4) Further analysis should concentrate on more subtle non-parametric analytical techniques like Owen's empirical likelihood ratio [22] to obtain more accurate bounds and regions of confidence.
5) Similar approach may be used in the other domains of materials science analysis where the normality assumptions is rather weakly met e.g. embedded systems [32], piezoelectric actuators [33-34], conical shells [35] and specific properties of designed materials [36-39].

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