DETERMINATION OF THE CRITICAL STRAIN FOR THE ONSET OF DYNAMIC RECRYSTALLIZATION OF C45 CARBON STEEL USING FLOW STRESS MODELS

OPĚLA Petr[1], SCHINDLER Ivo[1], RUSZ Stanislav[1], VANČURA Filip[2]

[1]*VSB - Technical University of Ostrava, Ostrava, Czech Republic, EU*
petr.opela@vsb.cz, ivo.schindler@vsb.cz
[2]*Kovárna VIVA a.s., Zlín, Czech Republic, EU,*
filip.vancura@viva.cz

Abstract

The critical strain for initiation of dynamic recrystallization is an important value in case of thermomechanical process design. In this research, different models have been utilized to describe hot compression flow curves of C45 carbon steel up to the peak point. Further, equations derived from these models were used to calculate the critical strain in wide range of thermomechanical conditions. Thus calculated values of the critical strain are function of the peak strain and of other deformation parameters. The calculations showed comparable results of almost each derived equation.

Keywords: Hot deformation, flow curve modeling, onset of dynamic recrystallization

1. INTRODUCTION

Dynamic recrystallization (DRX) plays a substantial role in case of controlling microstructural evolution and mechanical properties during hot forming of metals with low and medium stacking fault energy (SFE). The DRX process is initiated if a certain value of strain is reached. This value is known as the critical strain for the onset of DRX [1]. The critical strain for the onset of DRX is a crucial knowledge in thermomechanical process design [2]. Its value can be estimated metallographically. This evaluation is, however, time-consuming and the results are not precise [3]. The critical strain can be also calculated under different thermomechanical conditions using aptly derived equations. These equations can be derived from well-known flow stress models using the second derivative of work hardening rate with respect to stress [4]. Thus obtained values of the critical strain are function of the peak strain and other parameters of an original flow stress model [5]. Several equations have been derived by this approach in recent years. The main aim of the paper is to utilize these equations to estimate the critical strain of C45 carbon steel in wide range of thermomechanical condition.

2. CHARACTERIZATION OF CRITICAL STRAIN

In metals with a low or medium SFE, dynamic recrystallization takes place during deformation when a critical strain, ε_c (-), is reached. This value can be demonstrated on the typical DRX flow curve - see **Figure 1**. The work hardening and dislocation density increase with increasing strain, so, the flow stress is increasing (see region I.). When the critical strain is reached, new grains are nucleated at the old grain boundaries and newly developed high-angle boundaries grow. Nevertheless, the dislocation density in other areas of deformed material is still rises. Consequently, the flow stress increases but the increasing rate continuously declines (see region II.). Finally, the flow stress takes its maximum value (peak point). Then, the rate of softening mechanism of DRX prevails over the work hardening - the flow stress is decreasing (see region III.). The equilibrium between the work hardening and softening takes place at high strains, i.e. steady-state is occurred (see region IV.) [2, 4, 5].

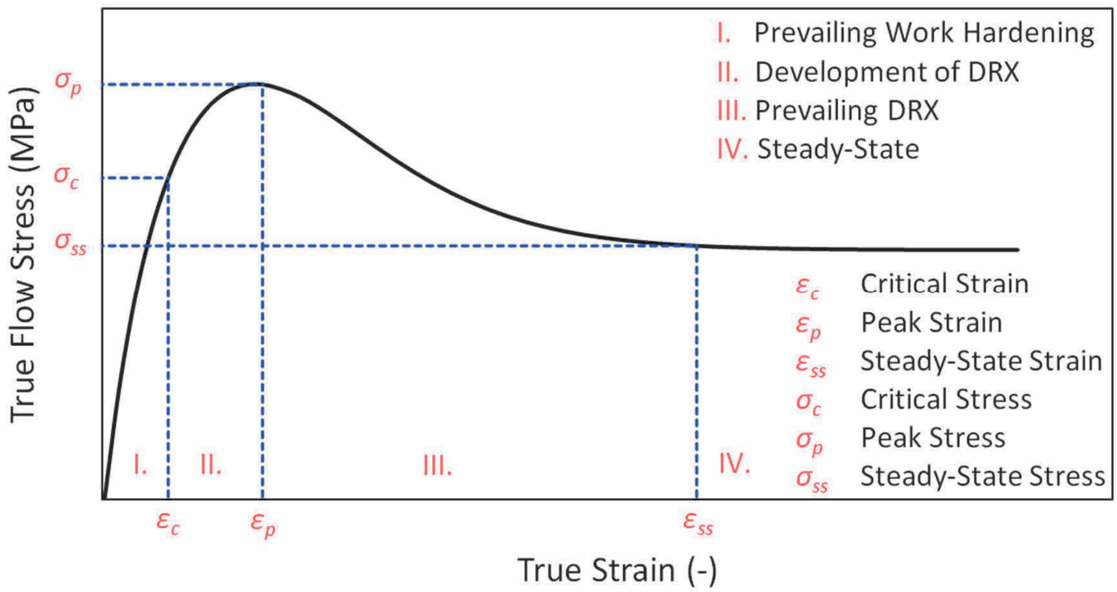

Figure 1 Schematic flow curve at high temperature for DRX [2]

3. HOT FLOW CURVE MODELING UP TO THE PEAK POINT

Flow curves of metals having low to medium SFE illustrate obvious DRX behavior [2] (**Figure 1**). These curves can be described in a wide range of thermomechanical conditions (i.e. strain, strain rate and temperature) by various flow stress models. A. Cingara and H. J. McQueen described flow curves up to the peak point (i.e. up to the strain of ε_p) by Eq. (1) [6]. S. Solhjoo also derived some equations to describe these curves - Solhjoo(A) Eq. (2) [4], Solhjoo(B) Eq. (3) [5] and Solhjoo(C) Eq. (4) [7]:

$$\sigma = \sigma_p \cdot \left[\frac{\varepsilon}{\varepsilon_p} \cdot \exp\left(1 - \frac{\varepsilon}{\varepsilon_p}\right)\right]^n \quad (1)$$

$$\sigma = \sigma_p \cdot \left[\frac{\varepsilon}{\varepsilon_p} \cdot \left(2 - \frac{\varepsilon}{\varepsilon_p}\right)\right]^n \quad (2)$$

$$\sigma = \sigma_p \cdot \left[\sin\left(\frac{\varepsilon}{\varepsilon_p} \cdot \frac{\pi}{2}\right)\right]^n \quad (3)$$

$$\sigma = \sigma_0 + \left(\sigma_p - \sigma_0\right) \cdot \left[\tanh\left(2 \cdot K \cdot \frac{\varepsilon}{\varepsilon_p}\right)\right]^n \quad (4)$$

In these equations, σ (MPa) is true flow stress, ε (-) true strain, and the subscript p corresponds to the peak coordinates. Parameter σ_0 (MPa) is the value of initial true flow stress. The parameters n (-) and K (-) represent strain hardening exponent and coefficient [4-6]. Most of the parameters of the above presented models exhibit a temperature and strain rate dependence. Mutual effect of these variables can be expressed by the Zener-Hollomon parameter, Z (s^{-1}) [8]:

$$Z = \dot{\varepsilon} \cdot \exp\left(\frac{Q}{R \cdot T}\right) \quad (5)$$

where $\dot{\varepsilon}$ (s^{-1}) is the strain rate, T (K) is the temperature, R (8.314 $J \cdot K^{-1} \cdot mol^{-1}$) is the universal gas constant, and Q ($J \cdot mol^{-1}$) is the hot deformation activation energy. The effective calculation of Q was in detail described for example in [9, 10].

The peak strain, ε_p (-), and the corresponding peak stress, σ_p (MPa), belong to the temperature and strain rate dependent parameters. Thus, their values could be calculated with respect to the Zener-Hollomon parameter Z as follows from the Eq. (6) and (7) [10]:

$$\varepsilon_p = a \cdot Z^b \tag{6}$$

$$\sigma_p = \frac{1}{\alpha} \cdot \operatorname{arcsinh} \sqrt[n]{\frac{Z}{A}} \tag{7}$$

where A (s^{-1}), a (-), b (-), n (-) and α (MPa^{-1}) are material constants. The calculation of these constants is in detail discussed in [10].

The strain hardening exponent, n, can be estimated for each combination of temperature and strain rate as the slope of the line from the logarithmic form of the Eqs. (1), (2) and (3). Then, the following equation can be used to describe the strain hardening exponent, n, with respect to the thermomechanical conditions [4-6]:

$$n = n_1 \cdot Z^{n_2} \tag{8}$$

In Eq. (8), n_1 (s) and n_2 (-) are material constants and can be calculated as the slope of the line and intercept of the logarithmic form of the Eq. (8), i.e. ln n vs. ln Z.

It should be noted, the strain hardening exponent, n, and coefficient, K, in the Eq. (4) are interdependent. Moreover, it was found out that these parameters are independent on the thermomechanical conditions. Their estimation is given by the iterative method in [7].

4. CRITICAL STRAIN FOR THE ONSET OF DYNAMIC RECRYSTALLIZATION

The critical strain for the onset of DRX can be determined metallographically. Nevertheless, this method is protracted and the results are not accurate [3]. Another possibility is to utilize the mathematical approach. This is based on analysis of the work hardening rate (θ) vs. stress (σ) curves up to the peak point. The work hardening rate, θ (MPa), is defined as derivative of $d\sigma / d\varepsilon$. According to Ryan and McQueen observation, an inflection in the θ - σ curve corresponds to DRX initiation [11]. Poliak and Jonas [3] demonstrated that the minimum point of $d\theta / d\sigma$ - σ curve represents the onset of DRX. From the mathematical viewpoint, the minimum point of $d\theta / d\sigma$ - σ curve is a null value of the second derivative of θ with respect to σ, and denote the inflection of θ - σ curve [12]. It should be mentioned that the inflection point of the θ - σ curve is the same as the inflection point of the ln θ - ε curve. These findings can be formulated by the following relationship [13]:

$$\left.\frac{\partial^2 \theta}{\partial \sigma^2}\right|_{\sigma=\sigma_c} = \left.\frac{\partial^2 \ln \theta}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_c} = 0 \tag{9}$$

Equations of the critical strain for the onset of DRX, ε_c, were derived by solving the Eq. (9) using the Eqs. (1), (2), (3) and (4). Derived equations are function of the peak strain, ε_p, and strain hardening exponent, n, or also strain hardening coefficient, K. The resulting expressions are named after the original flow stress model from which are derived, i.e. Cingara-McQueen Eq. (10) [2], Solhjoo(A) Eq. (11) [4], Solhjoo(B) Eq. (12) [5] and Solhjoo(C) Eq. (13) [7]:

$$\varepsilon_c = \varepsilon_p \cdot \frac{\sqrt{1-n}+n-1}{n} \tag{10}$$

$$\varepsilon_c = \varepsilon_p \cdot \left[1 - \frac{1}{2} \cdot \sqrt{4 - 2 \cdot \frac{6 \cdot (n-1) + 2 \cdot \sqrt{n^2 - 6 \cdot n + 5}}{2 \cdot n - 1}}\right] \tag{11}$$

$$\varepsilon_c = \varepsilon_p \cdot \frac{2}{\pi} \cdot \arctan\left(\sqrt{1-n}\right) \tag{12}$$

$$\varepsilon_c = \varepsilon_p \cdot \frac{1}{2 \cdot K} \cdot \operatorname{arctanh}\left(\sqrt{\frac{1-n}{1+n}}\right) \tag{13}$$

As a result of the above mentioned facts, if the flow curves description is available in the form of Eq. (1), (2), (3) or (4), the exact value of the critical strain, ε_c (-), could be calculated directly from Eq. (10), (11), (12) or (13). It can be noted, the values of the critical stress, σ_c (MPa), can be then calculated from Eq. (1), (2), (3) or (4) by substituting the value of strain, ε, by the corresponding value of the critical strain, ε_c.

5. RESULTS AND DISCUSSION

The above derived relationships were used to calculate the critical strain for the onset of DRX of the C45 medium-carbon steel in wide range of thermomechanical conditions. The experimental flow curves of the examined steel were obtained by the series of uniaxial hot compression tests. This experiment was described previously in the literature [14]. The result of these tests is the set of twenty flow curves acquired at the temperatures of 1280 °C; 1200 °C, 1100 °C, 1000 °C and 900 °C and the strain rates of 0.1 s^{-1}, 1 s^{-1}, 10 s^{-1} and 100 s^{-1}. Each test was performed for the values of true strain up to the 1.0. These experimental flow curves were analyzed and the flow curve description up to the peak point was assembled. As the very first step, the values of ε_p and σ_p were calculated for all sets of thermomechanical circumstances utilizing the Eqs. (5), (6) and (7). All required material constants of these equations are presented in **Table 1**.

Table 1 Material constants of the peak point description

A (s^{-1})	a (-)	b (-)	n (-)	Q (kJ·mol^{-1})	α (MPa^{-1})
$4\cdot10^{11}$	$3\cdot10^{-3}$	0.2	4.3	292	$9.5\cdot10^{-3}$

As the second step, the values of the strain hardening exponent, n, of the equations (1), (2) and (3) were determined for all sets of thermomechanical conditions using the Eq. (8). The necessary material constants of this equation are given in **Table 2**. The strain hardening exponent, n, and strain hardening coefficient, K, of the Eq. (4) are independent on the thermomechanical conditions, so they were obtained directly from the Eq. (4) by the iterative method [7]. The values of n and K of the Eq. (4) were calculated to be 0.86 and 1.53, respectively.

Table 2 Material constants of the strain hardening exponent

	Cingara-McQueen	Solhjoo(A)	Solhjoo(B)
n_1 (-)	11.20	7.05	5.63
n_2 (-)	− 0.14	− 0.13	− 0.12

Using the Eqs. (1), (2), (3) and (4), flow stress is calculated for all different thermomechanical conditions and compared with the experimental results. Example of this comparison is shoved in **Figure 2**.

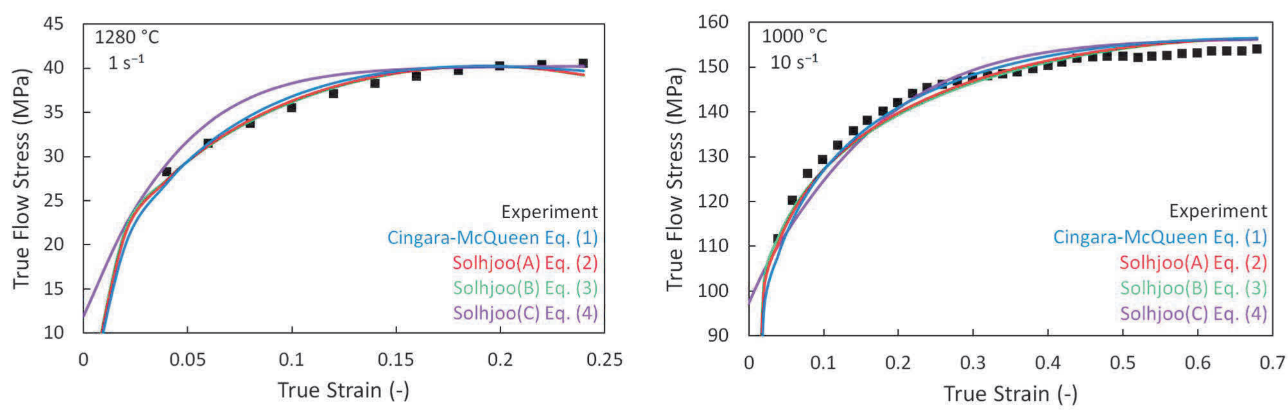

Figure 2 Comparison between experimental and predicted flow curves of the C45 carbon steel

The reliability of the flow curve description was assessed using average absolute relative error, AARE, - see **Figure 3**. The formulae of the AARE (%) can be written as follows [15]:

$$\mathrm{AARE} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left| \frac{\sigma_i - \sigma(\varepsilon_i)}{\sigma_i} \right| \qquad (14)$$

In Eq. (14), n (-) represents the number of data points involved in calculations, σ_i (MPa) is the target value and $\sigma(\varepsilon_i)$ (MPa) is the model output, i.e. experimental flow stresses and predicted one, respectively. The **Figures 2** and **3** imply the high accuracy and comparability of the proposed models. However, lower reliability was observed in case of the Solhjoo(C) model, i.e. Eq. (4).

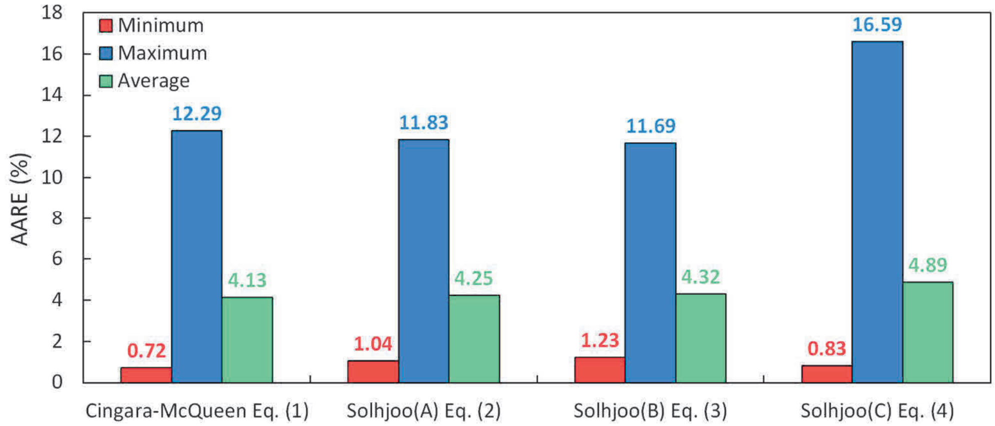

Figure 3 Average Absolute Relative Error (AARE) of the utilized flow stress models

Using the Eqs. (10), (11), (12) and (13), the critical strain for the onset of DRX has been calculated for all different thermomechanical conditions. The obtained results are related to the peak strain in **Figure 4**.

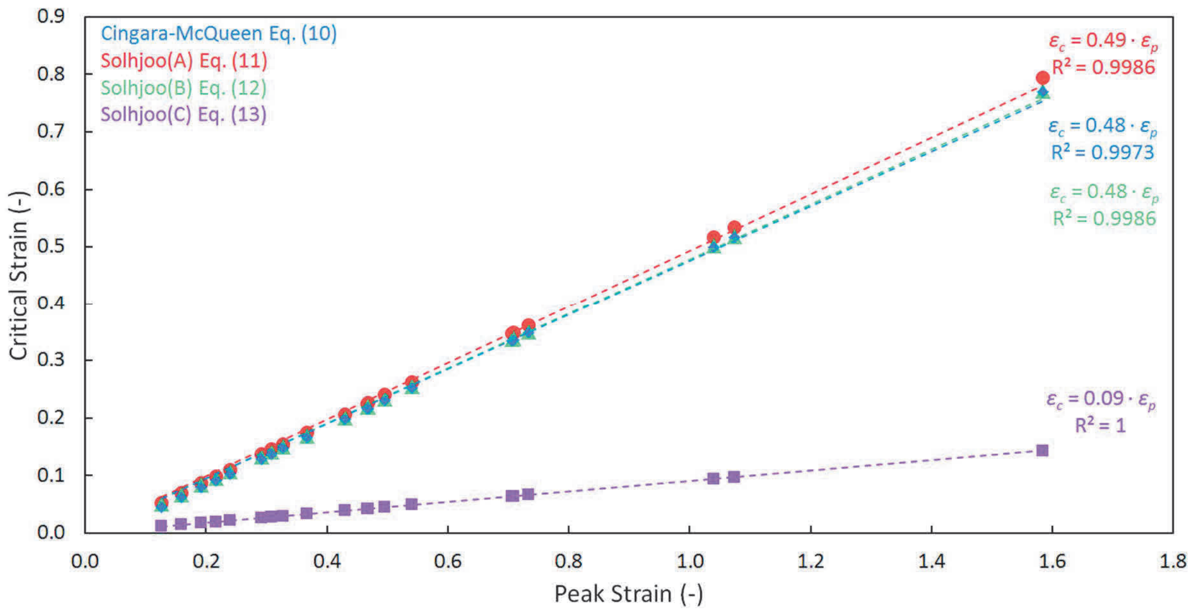

Figure 4 Relation between the critical strain, ε_c, and the peak strain, ε_p, of the C45 medium-carbon steel

The **Figure 4** clearly shows that almost each derived equation of the critical strain provides highly comparable results. Nevertheless, the Eq. (13), derived from the Solhjoo(C) model (4), offers exceedingly different values of the critical strain. However, this contradiction is not with respect to the flow curve comparison (see **Figures 1** and **2**) so astonishing. The Solhjoo(C) model (4) provides lower accuracy of the flow curve description.

Thereupon, inaccurate resulting values of the critical strain should be expected. The values of the critical strain of different steels were calculated and reported by several researchers in previous papers. The reported values are varying roughly in the range of $0.4 \cdot \varepsilon_p$ - $0.6 \cdot \varepsilon_p$ [2, 4, 7, 13]. So, the values calculated by the equations (10), (11) and (12) could be supposed to be utilizable.

6. CONCLUSION

The values of the critical strain, ε_c, for the onset of dynamic recrystallization of C45 medium-carbon steel were determined. This determination has been done in the wide temperature range of 900 °C - 1280 °C and the strain rate range of 0.1 s^{-1} - 100 s^{-1}. For this purpose, several mathematical models were utilized to directly calculate the values of ε_c. Nevertheless, one of these models was found to be less appropriate. Remaining models provide comparable results. The calculated values of the critical strain were related to the peak strain, ε_p. The appropriate results of the critical strain, ε_c, were found to be $0.48 \cdot \varepsilon_p$ and $0.49 \cdot \varepsilon_p$.

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