STATISTICAL CONTROL OF A MOULDING PROCESS

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Abstract

The paper deals with the problem of statistical process control (SPC) applied on the process of moulding tin coated cold rolled steel. The process level fluctuates around the historical mean, subgroup averages in the X-bar control chart show an extensive variation and as a result, frequent signals occur. Considering the high process capability, it would be pointless to seek assignable causes and take measures to remove this variation at the current state of the art; it is regarded an inherent part of the process. From this reason the classical X-bar control chart cannot be used. The solution consists in relaxing control limits so that only major shifts of the process mean are signalled. Several approaches based on a chosen time-dependent distribution model of the process are considered in the paper. Methods of the model identification and formulas for the construction of extended control limits are applied on two controlled quality characteristics, the planarity and length of moulded frames to show calculations for both one-sided and two-sided specifications. Moreover, calculation of capability and performance indices for the time-dependent models is discussed.

Keywords: Acceptance control chart, extended limits, modified control chart, process performance gaps

1. INTRODUCTION

Statistical process control is widely used in industry and the Shewhart chart is the most known tool of SPC. Basically, the aim of the X-bar chart is to control the process mean stability across the time. Originally, the aim of the control chart was to achieve continuous improvement in quality. However, as the quality of some processes improved, the process variation became so small compared to the specified tolerance limits, than further improvement does not seem economical. In some applications, minor shifts of the process mean are not considered a reason for intervention in the process. In this sense, a lot of redundant signals occur due to relatively narrow control limits based on a small process σ. The moulding process analyzed in the paper is an example of such application. Then the control chart should be rather used to monitor the process.

In [1] several time-dependent process models are distinguished according to the stability of the process mean and instantaneous variation and according to the type of instantaneous and outcoming distribution. The non-constant process mean is typical for the type C processes. In this paper random changes of the process mean are analyzed and models C1 assuming the normal outcoming distribution and C2 with a non-normal outcoming distribution are considered. Model C3 with systematic changes of the process mean was applied in [2].

The problem of adjusting control limits was solved in literature and various methods were introduced. The chart with extended limits [3], modified control chart, see e.g. [4], acceptance control chart [5], [6], [7] are used in this paper. The effective application of these methods requires some statistical knowledge, e.g. of ANOVA, distribution model identification, and others. The aim of the paper is to design a suitable control chart for the selected moulding process.

Apart from the control chart design, the process performance evaluation is important. Several constructions of performance indices were designed [8] but only some of them seem to be justified and are used in this paper. While the difference between type C1 and C2 processes may not be important when the X-bar control chart limits are constructed, the careful model identification plays a key role in the process performance evaluation where individual observations are treated. Therefore a detailed process analysis, which is described in this paper, is a necessary condition for the correct estimate of performance indices.

2. METHODS

2.1. Process model identification

To choose a suitable time-dependent model, homogeneity of the process mean and variance must be assessed as well as the type of both the instantaneous and outcoming distribution. The analysis is based on the ANOVA model with random effects, which reflects possible random changes of the process mean,

$$x_{ij} = \mu + a_j + \varepsilon_{ij} \tag{1}$$

where x_{ij} is the ith observation in subgroup j, μ is the grand mean, $a_j \sim N(0, \sigma_A^2)$ is the random effect of subgroup j, and $\varepsilon_{ij} \sim N(0, \sigma^2)$ is the random error effect. The null hypothesis H_0: $\sigma_A^2 = 0$ states that the process mean is constant. The test statistic is $F = MSA/MSE$ (**Table 1**). Rejection of H_0 implies that the sample mean variation is significant and consequently, the classical control chart is unusable.

Table 1 ANOVA

Source of variation	Sum of Squares	Df	Mean Square	*F*-ratio
Subgroups	*SSA*	*k* - 1	*MSA* = *SSA*/(*k* - 1)	*MSA*/*MSE*
Residual	*SSE*	*k*(*n* - 1)	*MSE* = *SSE*/(*k*(*n* -1))	
Total	*SST*	*kn* - 1		

The F-test is quite robust and so the assumption of normality is not critical in ANOVA. The effects of departure from normality on X-bar charts are not serious unless the instantaneous distribution is extremely non-normal. However, the proper model identification plays a key role in the process performance evaluation. Both tests and graphical methods are used to check for normality: Anderson-Darling (AD), Ryan-Joiner (RJ), Kolmogorov-Smirnov (KS), Shapiro-Wilk (SW), tests of skewness and kurtosis, and probability plots. The null hypothesis represents the assumption of normality. If the *p*-value associated with the test statistic chosen is greater than 0.05 (or 0.1), the variable of interest is assumed to follow a normal distribution. The probability plot helps to assess the cause of possible rejection of the normality assumption.

2.2. Control charts with adjusted limits

In a more general meaning, a process may be considered in control if the process mean fluctuates between some specified limits. To distinguish special causes from the random mean fluctuation, the adjusted control limits must be wider than the classical 3-sigma limits. Several approaches can be used to adjust the X-bar control limits.

The *extended control limits* are constructed at 3-sigma distance outwards from the limits for the process mean, which are located at the distance of Δ from the centre line

$$UCL = \bar{\bar{x}} + \frac{3\hat{\sigma}}{\sqrt{n}} + \Delta \qquad LCL = \bar{\bar{x}} - \frac{3\hat{\sigma}}{\sqrt{n}} - \Delta \tag{2}$$

The choice of constant Δ can be based on technical or economic considerations. Here $\Delta = 1.5\hat{\sigma}_A$ was chosen [1], where σ_A^2 and σ^2 are the variance components from model (1) that measure the fluctuation of the mean and the instantaneous inherent variation, respectively. They are estimated from the ANOVA table (**Table 1**) by

$$\hat{\sigma}_A^2 = \frac{MSA - MSE}{n} \qquad \hat{\sigma}^2 = MSE \tag{3}$$

Another way consists in constructing 3-sigma limits for subgroup means directly, based on their observed variation. The subgroup means are assumed to follow normal distribution and the limits are

$$UCL = \hat{\mu} + 3\hat{\sigma}_{\bar{x}} \qquad LCL = \hat{\mu} - 3\hat{\sigma}_{\bar{x}} \tag{4}$$

where $\hat{\sigma}_{\bar{x}} = \overline{MR}_{\bar{x}} / 1.128$, $MR_j = \bar{x}_j - \bar{x}_{j-1}$ for j = 2, 3, ..., k. Other estimates are also possible, see [9], [10].

Completely different approach is based on tolerance limits *USL*, *LSL* and a specified fraction of nonconforming units. Either the limits for the process mean are chosen to ensure the fraction of nonconforming of at most p_A with the type I error risk of α and the control limits are determined by

$$UCL = USL - u_{1-p_A}\hat{\sigma} + \frac{u_{1-\alpha}\hat{\sigma}}{\sqrt{n}} \qquad LCL = LSL + u_{1-p_A}\hat{\sigma} - \frac{u_{1-\alpha}\hat{\sigma}}{\sqrt{n}} \tag{5}$$

or the process fraction nonconforming p_R is specified together with the type II error risk of β, i.e.

$$UCL = USL - u_{1-p_R}\hat{\sigma} - \frac{u_{1-\beta}\hat{\sigma}}{\sqrt{n}} \qquad LCL = LSL + u_{1-p_R}\hat{\sigma} + \frac{u_{1-\beta}\hat{\sigma}}{\sqrt{n}} \tag{6}$$

where $\hat{\sigma} = \bar{R} / d_2$ and d_2 can be found for a given n in [11].

Frequently $u_{1-\alpha}$ = 3 and $u_{1-\beta}$ = 1.645 are chosen. The former option imitates 3-sigma limits in the Shewhart chart, the latter value corresponds to β = 0.05. As for u_{1-p_A} and u_{1-p_R}, the **Table 2** can be helpful; p_{out} denotes the fraction of nonconforming units under the condition that the mean process μ is shifted by 1.5σ from the centre towards *USL* or *LSL* and the process capability corresponds to C_p. The value close to the observed process C_p or C_{pk} should be chosen.

Table 2 Choice of *u*-scores

C_p	$u_{1-p_{out}}$	p_{out}
1.67	3.5	2.33 x 10^{-4}
2	4.5	3.398 x 10^{-6}
2.33	5.5	1.899 x 10^{-8}
2.67	6.5	4.016 x 10^{-11}
3	7.5	3.186 x 10^{-14}

2.3. Process performance

If the process mean fluctuates, the performance of the process is evaluated rather than its capability. Two situations are distinguished dependent on the form of the outcoming distribution.

For normal distribution the formulas

$$P_p = \frac{USL - LSL}{6\sigma_{B/W}} \qquad P_{pU} = \frac{USL - \mu}{3\sigma_{B/W}} \qquad P_{pL} = \frac{\mu - LSL}{3\sigma_{B/W}} \tag{7}$$

are used, where $\sigma_{B/W}$ is estimated using variance components, i.e. $\hat{\sigma}^2_{B/W} = \sigma^2_A + \sigma^2$. For a two-sided specification P_{pk} is determined as the minimum from P_{pU} and P_{pL}.

For a non-normal distribution the indices are determined according to [8]

$$P_p = \frac{USL - LSL}{x_{0.99865} - x_{0.00135}} \qquad P_{pU} = \frac{USL - x_{0.5}}{x_{0.99865} - x_{0.5}} \qquad P_{pL} = \frac{x_{0.5} - LSL}{x_{0.5} - x_{0.00135}} \tag{8}$$

where the percentiles $x_{0.99865}$, $x_{0.5}$ and $x_{0.00135}$ are estimated dependent on the identified distribution. Weibull or lognormal distributions are frequently used models. Otherwise, Pearson or Johnson curves may be useful.

3. MOULDING PROCESS

3.1. Description of the current process control

In one of the manufacturing processes in Laird company frames from tin-plated cold-rolled steel are moulded (**Figure 1**). In 4-hour intervals subgroups of 5 frames are taken from the process and the length, width and height are measured optically by Micro-Vu Excel 4520 with the scale resolution of 0.5 μm. Since the process level fluctuates over time, which is typical for moulding processes, subgroup averages plotted in the classical X-bar chart with 3-sigma control limits show an extensive variation and frequent signals occur, see **Figure 2** for *Length*. Owing to the fact that all observations are well inside the specified tolerance limits *USL* and *LSL*, workers responsible for the process use other action limits in the control chart. They are chosen empirically based on the tolerance limits. The action limits are drawn symmetrically around the centre line at target *T* and their distance equals 0.8(*USL* - *LSL*). Points outside these limits signal an assignable cause which has to be removed.

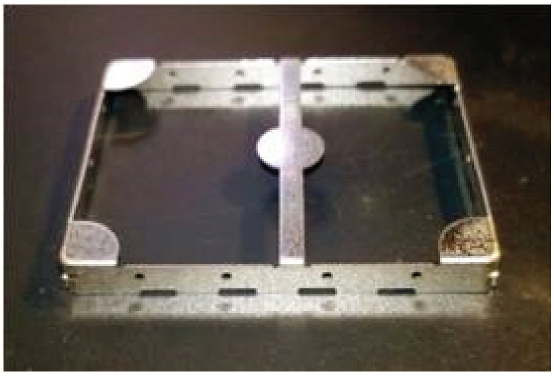


Figure 1 Frame moulded

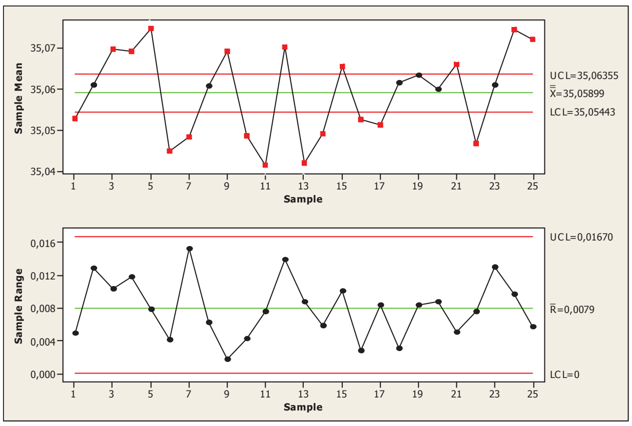


Figure 2 Shewhart control charts for *Length*

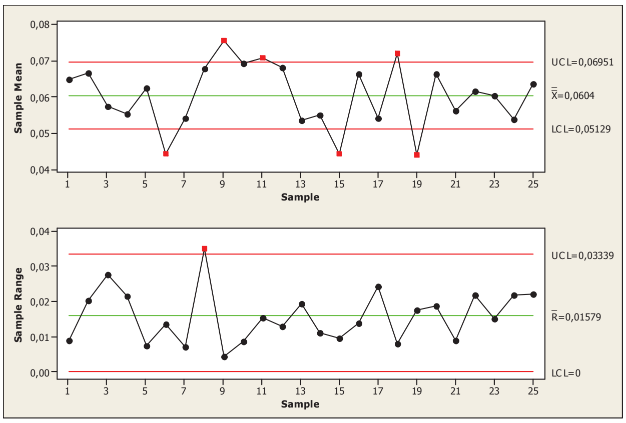


Figure 3 Shewhart control charts for *Planarity*

Except for the three dimensions, the planarity is calculated using height measurements at twelve specified points as the difference between the maximum and minimum observation. For variable *Planarity* only the upper tolerance limit *USL* is specified.

Since the use of the classical control chart is pointless, charts with adjusted control limits are designed in this paper to signal a change in the mean beyond the common fluctuation. Only two variables were chosen for illustration; the frame length with the specification of 35 ± 0.1 mm and the planarity with the upper tolerance limit of 0.12 mm. The analyzed part of 25 subgroups from the moulding process is considered in control in view of the process mean fluctuation.

The classical X-bar and R chart with 3-sigma limits are displayed in **Figures 2 and 3**. The red points in the X-bar chart lying outside the control limits are not considered to be the signals of special causes.

3.2. Control charts for *Length*

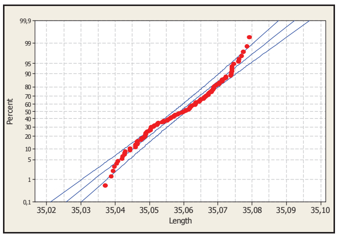
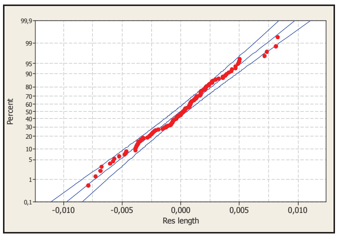
As confirmed by ANOVA (**Table 3**), the variation of the process mean is highly significant (the *p*-value less than 0.001).

Table 3 ANOVA for Length

Source	Sum of Squares	Df	Mean Square	*F*-Ratio	*P*-Value
Between groups	0.0131689	24	0.000548703	44.64	0.0000
Within groups	0.00122915	100	0.0000122915		
Total (Corr.)	0.014398	124			

Most normality tests in **Table 4** indicate a non-normal outcoming distribution (*p*-value < 0.05); based on the last two rows, it is the sample kurtosis which does not correspond to a normal distribution, see also the probability plot next to the *p*-values. The instantaneous distribution examined through residuals from ANOVA can be assumed normal (almost all *p*-values are greater than 0.1), see also the other probability plot.

Table 4 Checking for normality of *Length*

Outcoming distribution			Instantaneous distribution		
Test	***P*-value**	**Probability plot**	**Test**	***P*-value**	**Probability plot**
AD	< 0.005		AD	0.133	
RJ	0.021		RJ	> 0.1	
KS	0.039		KS	0.085	
SW	0.000		SW	0.350	
Skewness	0.741		Skewness	0.906	
Kurtosis	0.000		Kurtosis	0.474	

The adjusted control limits obtained by different methods together with the values needed for their calculation are displayed in **Table 5** and in **Figures 4 and 5.** For the modified limits the value of $u_{1\text{-}pA}$ corresponding to C_p = 2.33 was chosen, for the acceptance control chart $u_{1\text{-}pR}$ relates to C_p = 1.67 (**Table 2**).

Table 5 Adjusted control limits for *Length*

Method	Input values	*LCL*	*UCL*
ANOVA	$\hat{\sigma}_A^2 = 1.07 \times 10^{-4},\ \hat{\sigma}^2 = 1.23 \times 10^{-5}$	35.039	35.079
Sample means	$\hat{\sigma}_{\bar{x}} = 0.00997$	35.029	35.089
Modified	$u_{1-p_A} = 5.5,\quad u_{1-\alpha} = 3$	34.921	35.079
Acceptance	$u_{1-p_R} = 3.5,\quad u_{1-\beta} = 1.645$	34.927	35.073

As can be seen from **Figure 4**, the extended limits are adjusted to the observed subgroup mean variation, differently from the limits of both modified and acceptance control chart (**Figure 5**) that are derived from the target value and specified tolerance limits *USL* and *LSL*. All pairs of limits are wide enough for observed subgroup means to appear within these limits. The process is stable in a more general sense but obviously, its centre line is quite distant from the target (**Figure 5**).

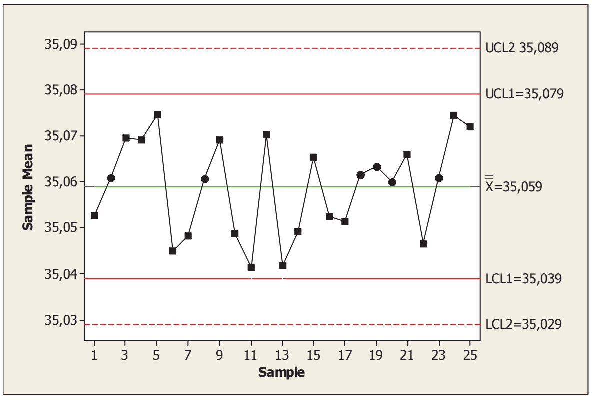


Figure 4 Chart with extended limits

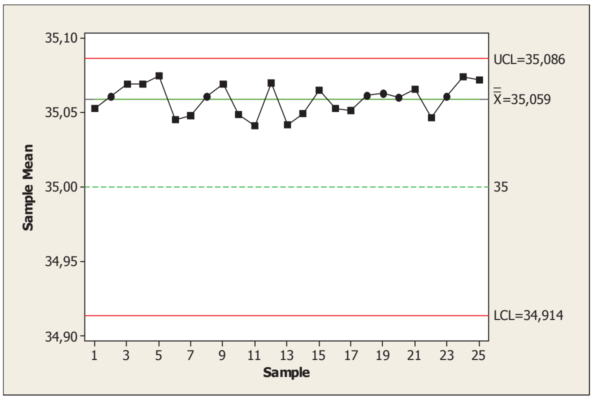


Figure 5 Modified/acceptance control chart

3.3. Process performance for *Length*

With regard to the results in **Table 4**, the outcoming distribution cannot be considered normal. Since no known theoretical distribution could be identified, the Johnson transformation was used. The transformation of type SB determined in Minitab is described by the equation

$$Y = -0.0952 + 0.8485 \cdot \ln\left(\frac{X - 35.0356}{35.0803 - X}\right)$$

The percentiles $\hat{x}_{0.00135}$ = 35.0365, $\hat{x}_{0.5}$ = 35.05 and $\hat{x}_{0.99865}$ = 35.0766 needed in (8) were obtained by the inverse transformation of $\hat{y}_{0.00135} = \bar{y} - 3s_y$, $\hat{y}_{0.5} = \bar{y}$ and $\hat{y}_{0.99865} = \bar{y} + 3s_y$ using the Solver Tool in Excel, where $\bar{y}$ and s_y denote the sample mean and sample standard deviation of the normally distributed transformed variable Y. For comparison, the performance indices under the normality assumption are calculated ($\hat{\sigma}^2_{B/W}$ = 0.0109).

Table 6 Process performance indices for *Length*

Method	$\hat{P}_p$	$\hat{P}_{pL}$	$\hat{P}_{pU}$	$\hat{P}_{pk}$
Johnson transformation	4.985	11.124	1.878	1.878
Formulas (7)	3.048	4.847	1.250	1.250

3.4. Control charts and the performance index for *Planarity*

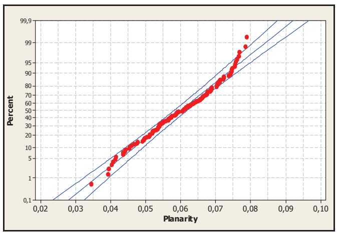
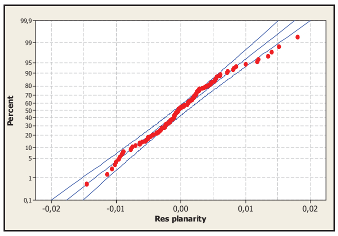
The variation of the process mean is highly significant, see **Table 7** (8. subgroup was dropped due to its excessive range, see **Figure 3**).

The results of normality tests for the outcoming distribution (**Table 8**) are rather ambiguous. Apparently the kurtosis differs from a normal distribution, see the probability plot. The instantaneous distribution of *Planarity* can be considered normal, based on both the *p*-values and probability plot in the right part of **Table 8**.

Table 7 ANOVA for *Planarity*

Source	Sum of Squares	Df	Mean Square	*F*-Ratio	*P*-Value
Between groups	0.00892871	23	0.000388205	9.68	0.0000
Within groups	0.00384908	96	0.0000400945		
Total (Corr.)	0.0127778	119			

Table 8 Checking for normality of *Planarity*

Outcoming distribution			Instantaneous distribution		
Test	*P*-value	Probability plot	Test	*P*-value	Probability plot
AD	0.122		AD	0.047	
RJ	> 0.1		RJ	0.055	
KS	> 0.15		KS	0.067	
SW	0.016		SW	0.331	
Skewness	0.480		Skewness	0.228	
Kurtosis	0.015		Kurtosis	0.094	

The adjusted control limits for *Planarity* obtained by different methods together with the values needed for their calculation are displayed in **Table 9** and drawn in **Figure 6**. Because of the one-sided specification, only the upper control limit is of interest. Both formulas (7) and (8) were used to evaluate the performance of *Planarity*. Under the normality assumption $\hat{P}_{pU}$ = 1.907, the use of the Johnson transformation described by *Y* = - 0.3236 + 1.12215·ln[(*X* - 0.0308)/(0.0834 - *X*)] gives $\hat{P}_{pU}$ = 2.301.

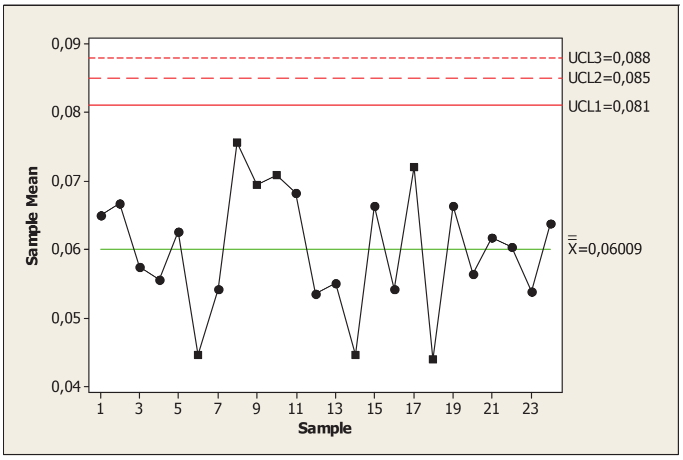


Figure 6 Chart with control limits from **Table 9**

Table 9 Adjusted control limit for *Planarity*

Control limits	Input values	*UCL*
ANOVA (1)	$\hat{\sigma}_A^2$ = 6.96 x 10^{-5}, $\hat{\sigma}^2$ = 4.01 x 10^{-5}	0.081
Sample means (3)	$\hat{\sigma}_{\bar{x}}$ = 0.0093	0.088
Modified (2)	u_{1-p_A} = 6.5, $u_{1-\alpha}$ = 3	0.085
Acceptance (2)	u_{1-p_R} = 4.5, $u_{1-\beta} = 1.645$	0.085

Note: The numbers in parentheses relate to the upper control limits in **Figure 6.** The modified and acceptance limits are the same here.

4. CONCLUSION

The use of adjusted control limits solves the problem of redundant signals that arise due to the process mean fluctuation. All the methods presented can be used to monitor processes of type C1 or C2 with the normal instantaneous distribution. But, as was noted above, the assumption of instantaneous normality is not strictly necessary. While the extended limits are suited to the current process observations with some acceptable mean fluctuation and seem to be suitable primarily in the phase I of SPC, when the process stability is of the main interest, the modified or acceptance limits are based on the given specification and apart from stability, they enable to control the process location. Both the modified and acceptance limits can be adjusted by the choice of an appropriate C_p (**Table 2**).

It appears that the difference between the performance index calculated under the assumption of normality and the index based on some more appropriate distribution model may be quite large if the underlying process is of type C2 and therefore the careful model identification is of great importance.

REFERENCES

[1] DIETRICH, E., SCHULZE, A. *Statistical Procedures for Machine and Process Qualification*. Cincinnati: Hanser, 2010. 396 p.

[2] JAROŠOVÁ, E., FOLTA, M. Statistical Control of a Process Subject to Tool Wear. In *METAL 2014: 23rd International Conference on Metallurgy and Materials*. Ostrava: TANGER, 2014, pp. 1464-1471.

[3] BISSELL, D. *Statistical Methods for SPC and TQM*. London: Chapman & Hall, 1994. 373 p.

[4] RYAN, T. P. *Statistical Methods for Quality Improvement*. J. New York: Wiley & Sons, 2011. 666 p.

[5] MOHAMMADIAN, F. Economic-Statistical Design of Acceptance Control Chart. *Quality and Reliability Engineering International*, 2012, vol. 29, no. 1, pp. 53-61.

[6] CHOU, C. Y. Acceptance Control Charts for Non-normal Data. *Journal of Applied Statistics*, 2005, vol. 32, no. 1, pp. 25-36.

[7] HOLMES, D., MERGEN, A. E. Building an Acceptance Chart around the Process Mean. 2010. Accessed from http://scholarworks.rit.edu/other/688.

[8] ISO 21747: Process performance and capability statistics for measured quality characteristics. ISO 2006.

[9] JAROŠOVÁ, E., NOSKIEVIČOVÁ, D. *Pokročilejší metody statistické regulace procesu.* Praha: Grada Publishing 2015. 290 p.

[10] KAYODE, S. A. Modified Simple Robust Control Chart Based on Median Absolute Deviation. *International Journal of Statistics and Probability*, 2012, vol. 1, no. 2, pp. 91-95.

[11] ISO 7870-2: Shewhart control charts. ISO 2013.