ASSESSMENT OF METALLURGICAL PROCESS CAPABILITY WITH MORE QUALITY CHARACTERISTICS

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Abstract

This paper presents a way to assess processes where more quality characteristics are observed. The method used concerns multivariate capability indices supported by graphical methods. The emphasis is placed on thorough testing of conditions that validate such procedure. The methodology is illustrated on metallurgical data related to a casting which is assessed by two quality characteristics - ultimate tensile strength (X_1 in pascals) and yield strength (X_2 in pascals).

Keywords: Metallurgy, capability, normality, homogenity, outliers, centralization, process stability

1. INTRODUCTION

Technological processes are monitored and managed in pre-production and production stages where specific statistical tools are used. One of the most important tools are design of experiments utilized in pre-production stages, and control charts that are used in all production stages to monitor inputs, processes and outputs. In final stages, the process is assessed, above all, usually by means of univariate capability indices. These are a common part of process documentation, and many customers prescribe their values for the process owners. At the same time, however, the indices are often worked with without compliance with conditions that must be met in order for the values of the indices to represent reliably the process status. For a precise description of univariate capability indices, we may refer to [1], which contains 170 most frequent practical problems, 100 solved examples and a training software *Capa.*

In this paper we shall deal with more advanced methodology for process assessment, in which several quality characteristics are observed. We shall present multivariate indices and conditions under which the indices can be used reliably. The methology is illustrated on data from metallurgical industry, and casting, in particular, where two quality parameters were evaluated - ultimate tensile strength (X_1) and yield strength (X_2).

2. PROBLEM FORMULATION

For modelling purposes and process assessment, thirty castings were observed at a metallurgical company, and their chemical composition, tensile strength and yield strength were measured and analyzed. The final chemical composition of a cast is controlled in line with the organization's regulations. Trials are performed on cuts corresponding to the norm EN DIN 1563.

Target values for firmness characteristics are usually defined by customer. The tolerance limits are calculated as follows: the lower limit LSL is $LSL = T - k{\cdot}s$, the upper limit USL is $USL = T + k{\cdot}s$, where s is the standard deviation of the observed variable X_i; k is set at 4. In our case, for example, the target value T was 250 for the first variable, and so if $s_1 = 6.1$, $LSL_1 = 250 - 4{\cdot}6.1 = 225.6$ and $USL_1 = 250 + 4{\cdot}6.1 = 274.4$. Since all outputs are firmness characteristics, exceeding the upper limit is regarded as not achieving the target value only, not as an unacceptable defect.

The target values and tolerance limits for the two observed variables are in **Table 1**.

Table 1 Target values and tolerance limits for the variables X_1 and X_2

	X_1	X_2
LSL	225	362
USL	275	418
T	250	390

Control results for the variables are in **Table 2**.

Table 2 Control results for the quality characteristics X_1 and X_2

Run	X_1	X_2	Run	X_1	X_2
1	266	401	16	258	395
2	267	406	17	260	399
3	259	396	18	260	399
4	268	404	19	260	401
5	274	413	20	260	403
6	259	405	21	258	392
7	269	412	22	258	393
8	273	416	23	269	405
9	263	403	24	269	402
10	267	411	25	273	414
11	265	401	26	263	399
12	265	401	27	263	399
13	253	394	28	264	396
14	253	394	29	263	401
15	274	408	30	277	418

3. CONDITIONS FOR USING UNIVARIATE CAPABILITY INDICES

When using capability indices to assess process capability, it is necessary to verify whether certain obligatory conditions hold. These conditions include, in particular, the following: *process stability*, *independence in the data*, *no outliers in the data*, *correctly defined tolerance*, and for some indices, *normality* and *process centralization* (i.e. average of the quality characteristic Xp equals T, Xp = T) are also required.

The conditions are required both for univariate and multivariate indices, and are verified by statistical tests. It is quite convenient to accompany the numerical verification methods with graphical tools, as well. In what follows, importance of the conditions is explained.

Process stability: This requirement means that the observed characteristic takes on values within specified limits of a control chart. Thus, control charts (CC) are the means to assess process stability. Many organizations, however, do not use them; in that case, a special test of process stability can be performed (in *Capa*, for example), whether one or more quality characteristics are evaluated.

Process stability is an essential condition. If the condition does not hold, the process is not under control, and therefore it cannot be evaluated (at a particular moment). If this is the case, the organization must adopt measures to stabilize the process and bring it under control. In our practical study, process stability was confirmed for both characteristics X_1 and X_2.

Normality and process centralization: For univariate indices, centralization, but also normality and potential occurrence of nonconform process outputs, when a piece of data lies outside the tolerance interval, can be judged upon on the basis of a graph. This can be seen in **Figure 1**, constructed for X_1.

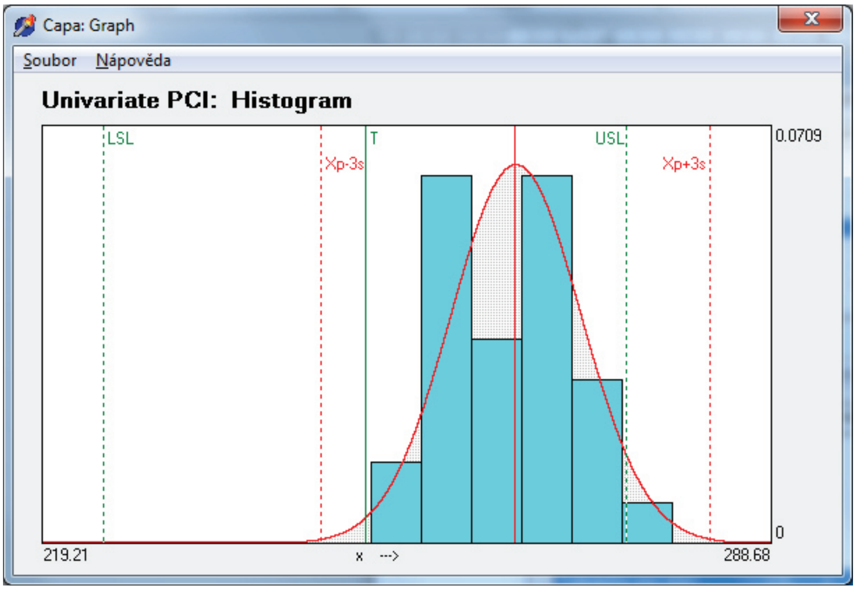

Figure 1 Graphical depiction of *normality* and *centralization* for the quality variable X_1

The figure shows the tolerance limits *USL* and *LSL*, the target value *T*, the average value of X_1 (the red vertical line passing through the vertex of the Gaussian curve) and an interval - average of $X_1 \pm 3s$, where 99.73 per cent of the values of X_1 occur. To make a judgement on whether the expected value (mean) of X_1 differs from *T*, a statistical test should be used. The shape of the histogram suggests that normality is a possibility here, although this should also be tested statistically.

As far as nonconformity is concerned, the process is shifted towards *USL*, it even exceeds the upper tolerance limit in some cases, and so, nonconform units (NC) do occur. Their proportion equals 0.0424, or 4.24% in per cent of the output, as calculated by *Capa*. To calculate the proportion by hand, the indices *CpL* and *CpU* can be used: NC = $\Phi(-3 \cdot CpU) + \Phi(-3 \cdot CpL) = 0.04239$, where Φ is the probability distribution function of the standard normal distribution.

To test univariate normality, many software packages can be used. The procedure is a standard part of their contents. To use our study as an example, univariate normality was tested in Statgraphics and Capa, and the results of the test carried out for both observed variables are in **Table 3**. The symbols *T* and *W* represent corresponding test criteria, *K* stands for the critical value.

Table 3 Test of univariate normality

X_1	X_1	X_2	X_2
Capa	Statgraphics	Capa	Statgraphics
$T = z = 0.059$	$W = 0.965$	$T = z = 1.072$	$W = 0.944$
$K = SW = 1.644$	p-val=0.47	$K = SW = 1.644$	p-val=0.14
Normality YES	Normality YES	Normality YES	Normality YES

Testing centralization means testing whether the target value T_1 defined for X_1 (or T_2 defined for X_2) equals the expected value of X_1 (or X_2). At a five per cent significance level, performing the test for X_1 gives the following results: $T_1 = 250$, average of $X_1 = 264.3$; test criterion $TK = 12.68$; critical value $SW = 2.04$. Since $T > K$, the hypothesis of centralized process is rejected. This conclusion is also reflected in smaller *Cpm* which equals 0.53. For the second variable, X_2, the test result is: $T_2 = 390$, average $X_2 = 402.7$; test criterion $TK = 9.90$; critical value $SW = 2.04$. Since $T > K$, the hypothesis of centralized process is rejected in the latter case, as well. This is reflected in *Cpm* which equals 0.64.

4. MULTIVARIATE CAPABILITY INDICES

Process stability: as in the univariate case, the stability may be confirmed or rejected with a control chart designed for more quality characteristics. This, however, isn't often available in companies, and so, as a substitute but correct solution, a statistical test can be used. Using *Capa* (see **Figure 2**), for instance, the test criteron $TK = 1.222$, while the critical value of the test $t = 7.815$. Since $T < t$, the process is stable. The test result (applies to all tests in the software) is also verbal - Process stability: Yes. Therefore, the most important condition for capability assessment is satisfied.

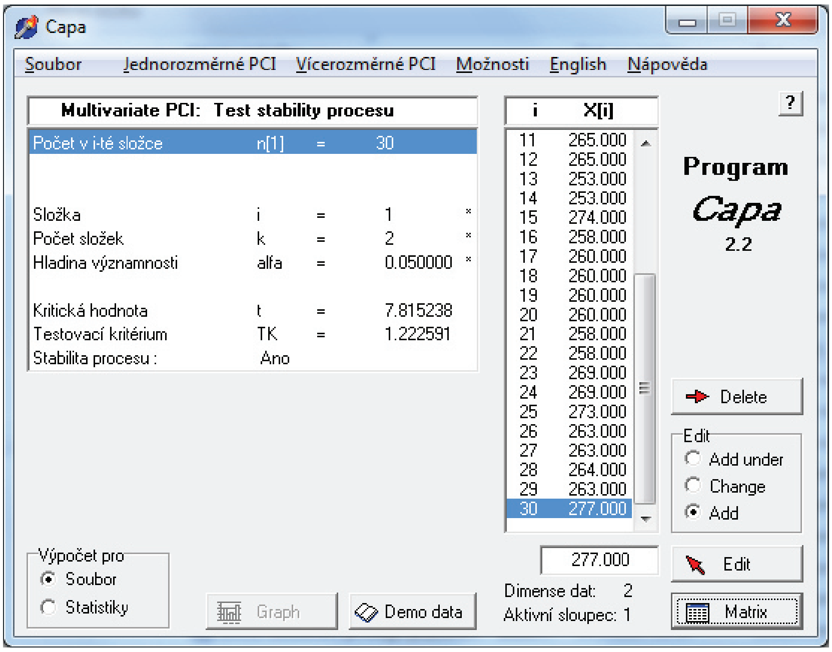

Figure 2 Test of multivariate stability

In **Figure 2** and other figures, the notation is:
Soubor = File,
Jednorozměrné = Univariate,
Vícerozměrné = Multivariate,
Možnosti = Options,
Nápověda = Help,
Složka = coordinate,
Počet složek = number of coordinates,
Hladina významnosti = significance level,
Kritická hodnota = critical value,
testovací kritérium = test criterion,
Stabilita procesu = process stability,
Toleranční mez = tolerance limit,
Dolní = lower,
Horní = upper,
Aritmetický průměr = mean,
Směrodatná odchylka = standard deviation.

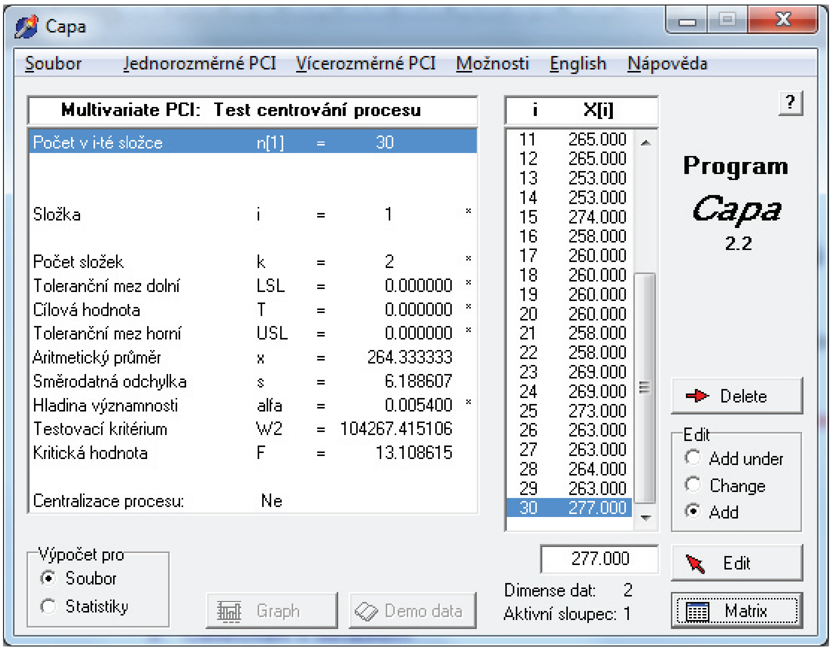

Figure 3 Centralization test for the vector (X_1, X_2)

Cases of instability are difficult to solve, as their causes must be revealed. In any case, it is meaningless to assess unstable processes with capability indices.

The following section presents evaluation of *multivariate normality* and *process centralization*. These are not conditions whose violation would exclude process assessment, nevertheless, their verification or rejection affects the subsequent selection of proper capability indices.

When two or more quality characteristics are observed, it is problematic to describe normality and centralization graphically. But then again, the only objective tool is a statistical test, after all.

Process centralization of the vector (X_1, X_2): in the bivariate case [2], when two quality variables are questioned, and their target values are denoted T_1 and T_2, respectively, while their expected values (means) are denoted PX_1 and PX_2, respectively, process centralization means that the points (T_1, T_2) and (PX_1, PX_2) are the same. The opposite case reflects a poorer quality, and results in a greater risk of exceeding tolerance limits and generating nonconforming products. In other words, financial losses are eventually incurred in such cases. A test of the centralization may be run in *Capa* (**Figure 3**).

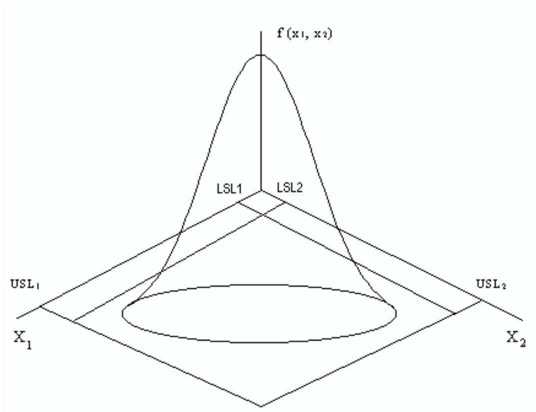

Figure 4 Normally distributed vector (X_1, X_2)

Normality of the vector (X_1, X_2): for the bivariate vector (X_1, X_2), normality is depicted in **Figure 4**. The vertical axis represents the values of the two-dimensional probability density function f(x_1, x_2), whose shape can be visualized by relative frequencies of occurence of the numerical pairs (x_1, x_2) obtained by sampling of the two characteristics in question.

For the normal case, the ellipse, lying in the tolerance rectangle with sides (LSL_1, USL_2) and (LSL_1, USL_2) and representing a large enough „mass“ under the density function, should not exceed the upper limits of the rectangle. Its centre should be as close to the target point (T_1, T_2) as possible (centralization).

Multivariate normality can be tested by comparing empirical skewness *g1m* (Multi Skewness) and empirical kurtosis *g2m* (Multi Kurtosis) with their theoretical counterparts for normal distribution. For our data, *g1m* = 0.356 and *g2m* = 5.948, which confirmed normality.

*Multivariate capability indices:*To assess process capability when more of its quality characteristics are observed, multivariate indices *MCp*, *MCpm* and *MCpk* are used. Conditions for their application are different, however. The indices *MCp* and *MCpm* require normal distribution and two-sided tolerance. For our study, the conditions are satisfied, and the two indices can therefore be calculated.

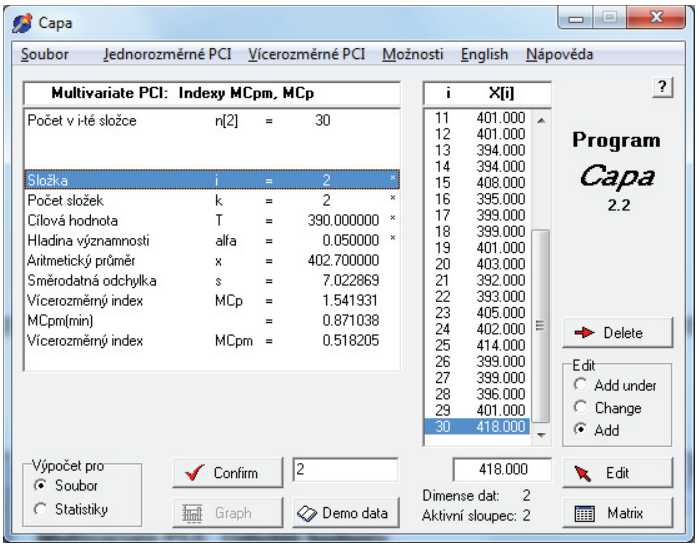

Figure 5 Calculation of multivariate indices *MCp* and *MCpm*

Using *Capa, MCp* = 1.5419, which is a good value. However, as in the case of the univariate index *Cp*, there is a specific condition of process centralization, which was not satisfied in our case. Thus, to make a proper judgement on the process, the *MCpm* index will be more suitable, since it doesn't require the condition of centralization. In the study, *MCpm* = 0.5182. The value is under the critical value 1.0, although any conclusion must be made after testing the significance of *MCpm*. To sum up the results so far obtained from the two indices: The process is not centred properly (low *MCpm*), but its variability is low as well, i.e. it doesn't vary much around its mean (high *MCp*). Therefore attention should be focused on centering the process.

Testing the indices: The indices *Cp* or *MCp* are usually evaluated by comparing their values with 1.0. Such an assessment is not possible in the case of *MCpm*, since even a value of *MCpm* below 1.0 can be significant. One has to find out a minimal required value *MCpm*(min). In our case, *MCpm*(min) = 0.871. Since *MCpm* = 0.518 < *MCpm*(min), the index is insufficient. The calculations, using *Capa*, are in **Figure 5**.

If the observed quality characteristics are not normally distributed, the *MCPk* index can be calculated. To do so, the proportion of nonconforming products must be available. The characteristics must also be independent. The index was not used in our study.

Table 4 lists the conditions that must hold for various multivariate capability indices which are to be used to assess a process with quality characteristics X_1, X_2, …, X_k.

Table 4 Conditions for multivariate capability indices

MCp, MCpm	***MCpk***
Normally distributed variables X_i's	Independent variables X_i's
Two-sided tolerance	Any distribution of X_i
The variables X_i's can be dependent	The X_i's can be attributes

The *MCp* index is defined as

$$MCp = \frac{\Pi_{i=1}^{k}(USL_i - LSL_i)}{\Pi_{i=1}^{k}(U_i - L_i)}, \tag{1}$$

where LSL_i and USL_i are limits of the tolerance interval for X_i.

The *MCpm* index is defined as

$$MCpm = \sqrt{\frac{n.k}{\sum_{i=1}^{n}(X_i - T)^T V^{-1}(X_i - T)}}, \tag{2}$$

where
$\boldsymbol{X}_i$ = the i-th column of the data matrix $\boldsymbol{X}$
$\boldsymbol{V}$ = variance matrix,
k = number of characteristics,
n = sample size

Outliers: In the multivariate case, it is also observed whether all characteristics take on values within their tolerance. **Figure 6** shows presence of outliers for the data under study. The software output shows that

Coordinate in tolerance: No
The vector in tolerance: No

„Coordinate" relates to a tested variable in the vector. In **Figure 6**, the first of the two characteristics is tested for outliers. In the software output, it is listed as the first term that must be entered in the program (the terms that must be supplied to the software are denoted with asterisk "*"). If a coordinate lies outside the tolerance, the entire vector is automatically regarded as being outside the tolerance.

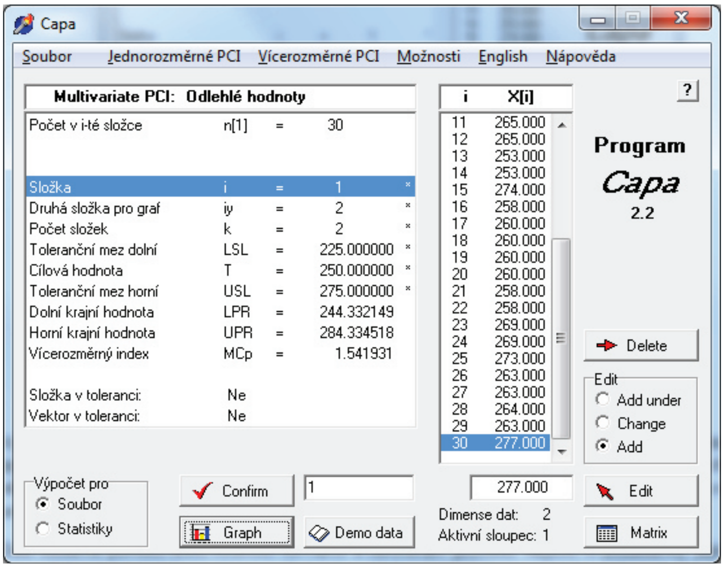

Figure 6 Outliers in the multivariate case

In the multivariate case, outliers are detected with the criterion *M* [3], which for the bivariate case equals

$$M = \max\left\{1, \frac{|U_1 - LSL_1|}{USL_1 - LSL_1}, \frac{|USL_1 - L_1|}{USL_1 - LSL_1}, \frac{|U_2 - LSL_2|}{USL_2 - LSL_2}, \frac{|USL_2 - L_2|}{USL_2 - LSL_2}\right\} \tag{3}$$

M should be smaller than 1 if there are no outliers.

5. CONCLUSION

Assessment of metallurgical processes with multivariate quality characteristics can be performed with multivariate capability indices, as in the case of one quality characteristic. The conditions for their use are virtually the same, but different statistical methods and special software is required to test the conditions. Satisfying the conditions is essential for the multivariate indices to be used for an objective metallurgical process assessment. Multivariate indices provide a more complex evaluation than univariate indices. Nevertheless, they should also be complemented with an economic process assessment (see [3], [4], [5] for instance), because organizations usually have two strategic objectives: maximal quality and minimal costs. This paper focused on the conditions that indices must satisfy to be of value, and presented a special software *Capa* that can handle these tasks even in the multivariate case.

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