# THE MATHEMATICAL MODEL DESCRIBING THE INFLUENCE OF CHOSEN PARAMETERS ON THE QUALITY OF FINAL PRODUCTS MANUFACTURED FROM TYPE 1.4307 STEEL

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**Abstract**

This article introduces a mathematical model describing how certain parameter changes influence the quality level of final products manufactured out of X2CrNi18-9 steel type 1.4307. Among the examined parameters are: batches of faulty steel sheets, processing not meeting desired standards, erroneously planned processing methods etc. The simulation results obtained with the use of the designed model determine the approach when making the choice on how to maintain the supply level for one of the logistic chains for the examined type of steel. The model utilizes Bernoulli, Poisson, Pascal probability distributions as codependent.

**Keywords:** Quality of finale products, mathematical model describing the quality

## 1. INTRODUCTION

The logistic chain for manufacturing metal products is a system comprised of multi-staged, divergent processing stages, which as a whole make up an added value. The beginning of the logistic chain marks the extraction of natural resources and shaping them into products matching the requirements of the final receiver through complex processing methods. Through the concept of the system theory, such a system may be called a grand and complex one [1], which may be considered a model of codependent isolated subsystems [2]. Dividing a complex system into subsystems facilitates undertaking various analysis and evaluations of the functioning of particular modules. It makes achieving the assumed goals effective and efficient [3]. The isolated subsystem may be considered independent and complex, which may be further decomposed into a number of subsystems. The number of levels a system may be broken down into in such a way can be agreed upon and depends on the criteria established. Dividing the system into subsystems allows precise identification of the operational task compilation, which are being evaluated. The system approach also requires the analysis of the environment and its impact on the analyzed system and its modules. The functionality and efficiency of each logistic chain link influences the overall productiveness of the entire system [4]. The fundamental mechanics powering the metallurgical production system is manufacturing products according to the 7R principle (right product, right quality, right quantity, right place, right price, right time, right customer). Therefore the main reason for the flow in a metallurgical logistic chain are the final recipient`s demands and the level of achieved goals (eg. efficiency, quality) has direct impact on the final product`s costs.

The main article aims at presenting an analyzed mathematical model showcasing quality issues probability in the final link of the metallurgical logistic chain. The analysis considered a chosen flow of a realistic object, in which various steel products are manufactured in accordance to the individual requirements of the recipient. The discussed company owns a modern machinery park, which influences executing special orders requiring the highest level of quality. Based on historical data, a material flow characterized by financial loses had been singled out. Defective final products were the reason for these financial loses. The most common cause for discarding a product as defective was damage done throughout various stages of the flow. **Figure 1** depicts the analyzed pattern of material circulation in a mixed (i.e. serial-parallel) structure.

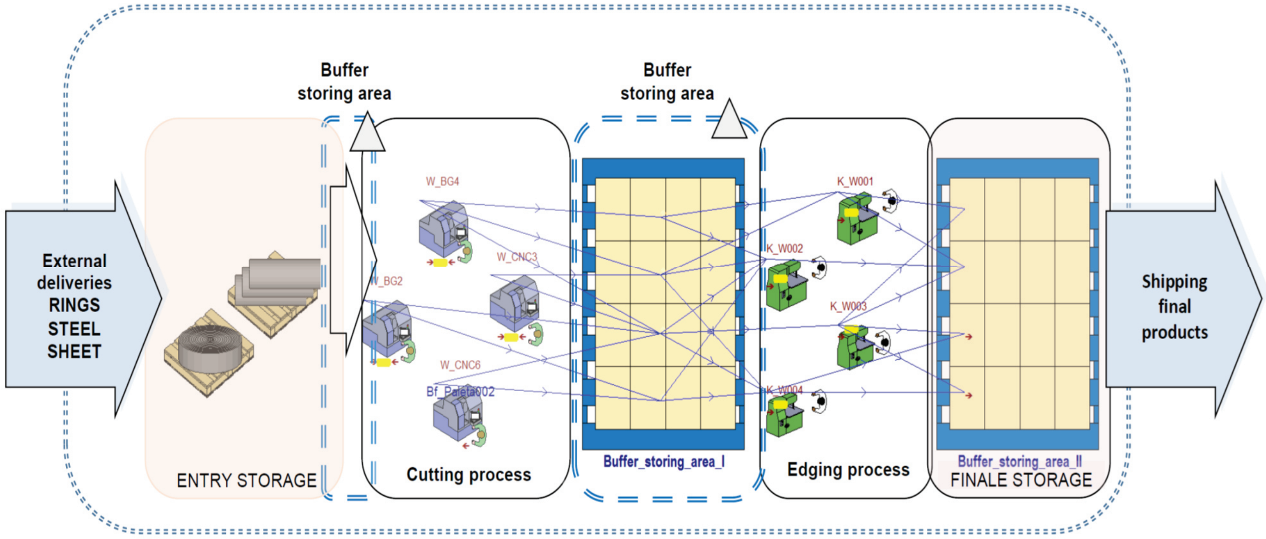


**Figure 1** Production system flow scheme

The figure depicts six areas of product flow. The model takes into account three main points generating quality errors, i.e. external deliveries, the cutting and edging processes. It is worth noting that manufacturing a certain type of product requires specific tasks, which may not be necessary in the case of other products. This is why the results obtained from the simulation utilizing the presented model are only true to the parameters of the analyzed layout.

## 2. THE MODEL DESCRIBING ON THE QUALITY OF FINAL PRODUCTS

The analyzed system had been decomposed to single flow streams as a result of a multi-level classification according to the following criteria:

1) Type of steel sheet - the analyzed system processes 39 various types of sheets, e.g. galvanized thermally or electrolytically, ferrytic (e.g.: 1.4016, 1.4509), austenitic (e.g.: 1.4301, 1.4404), and coated in all possible colours;
2) Sheet thickness: 0.5 mm, 0.8 mm, 1.0 mm, 1.5 mm and 2.0 mm;
3) Width: 550 mm, 750 mm, 1000 mm, 1151 mm, 1250 mm;
4) Length: 1100 mm, 1270 mm, 1400 mm, 1600 mm, 1900 mm, , 2500 mm, 3450 mm, 3800 mm.

Varying width and length of steel sheets is a result of optimizing the cutting process in order to minimize excess material being wasted. By multiplying factors depicted in points 1 through 4, more 8000 various flow streams may be distinguished. From these streams, about 1500 lines of products have been manufactured during the last year (2015) in the volume ranging from 1 to 2.7 million items. In a company a considerable group of piece production items can be distinguished, manufactured according to the demands of individual recipients, and series production (medium and big lot production). The processing actions had taken place in two main production subsystems: cutting and edging. Each of the aforementioned subsystems contains specific parameters of machinery resources, functionality state and dependency in regards to the remaining system`s modules. The cutting area is equipped with 3 CNC laser machines and 5 machine cutters, also known as hammer punches. The machine resources of the bending area are 9 presses of various bending width (ranging from 2000 to 4000 mm.) For each distinguished singular flow stream the product quality level had been calculated and the cost for eliminating inadequacies had been estimated. The highest costs had been generated for streams displaying the highest quality levels according to the SixSigma method, which was respectively 99.97 and 98.94 PPM (Percent Per Million). Achieving a high rate of satisfying products does not guarantee minimal financial investment on manufacturing defective products. The costs are influenced by a number of factors, such as: the type of steel sheets, their price, quality of received products, the quality level of certain stages not only concerning production but also storage, the cost of manufacturing some elements, the number of manufactured elements in a unit of time, DPU rate (Defects Per Unit) and finally the stage during which the defect had been detected, rendering the product unacceptable. In an effort to minimize the funds spent on eliminating quality issues, a mathematical model had been created to calculate the probability of inadequacies occurring based on the following parameters: quality of the delivered steel sheets, process quality of various stages, point of identifying the issue. The designed model is independent in regards to the total volume of manufactured assortment, therefore is applicable to piece production as well as big lot production. For the analysis the following steel sheet types have been chosen: X2CrNi18-9 (type 1.4307), dimensions 0.8 x 2600 x 1250 [mm x mm x mm] and 2.0 x 1300 x 1250 [mm x mm x mm], due to generating highest costs in eliminating errors. In the year 2015 the share of these two streams amounted to 12% of the entire costs contributed to eliminating quality issues. The model assumes the number of forms obtained from a single sheet which varies for sheets 0.8 thick between 1 to 300 pieces, for sheets 2.0 mm thick it varies from 1 to 100 pieces. The DPU rate for the front layer equals 0, while the rear layer is subject to individual (subjective) assessment of the quality inspector.

### 2.1. Mathematical model of the analyzed case

Discussing steel sheets type X2CrNi18-9 in two magnitudes: 0.8 x 2600 x 1250 [mm x mm x mm] and 2.0 x 1300 x 1250 [mm x mm x mm]. The descriptive portion of the model`s parameters utilizes index 0. For sheets 2.0 x 1300 x 1250 mm, while 2.0 is utilized for 2.0 x 1300 x 1250 mm sheets. The specified size is analyzed by applying 0.8 and 2.0 accordingly. The mathematical model describes three stages in which errors are generated: errors from suppliers, errors done within the cutting and edging stages.

The following markings had been introduced:
$X^{0.8}$ - the number of sheets in format 0.8 x 2600 x 1250 [mm x mm x mm]
$X^{2.0}$ - the number of sheets in format 2.0 x 1300 x 1250 [mm x mm x mm]
X - total number of sheets,

$$X = X^{0.8} + X^{2.0} \tag{1}$$

$p^{0.8}$ - the probability of damaging a singular sheet in format 0.8 x 2600 x 1250 [mm x mm x mm]
$p^{2.0}$ - the probability of damaging a singular sheet in format 2.0 x 1300 x 1250 [mm x mm x mm]

From historical data it may be concluded that the values $p^{0.8}$ and $p^{2.0}$ are minute, below 5%. The model assumes that the damage done to various sheets in format 0.8 is not linked to each other. The same conclusion has been drawn regarding sheets in format 2.0. Therefore we may assume that the total amount of damaged sheets in format 0.8 obeys the Poisson distribution with the parameter of $\lambda_1^{0.8}$ and the summary number of damages in format 2.0 also obeys the Poisson distribution with the parameter of $\lambda_1^{2.0}$ [5].

$N_1^{0.8}$ - the random variable describing the total number of damaged sheets in 0.8; $N_1^{0.8} \sim$ Poiss ($\lambda_1^{0.8}$)
$N_1^{2.0}$ - the random variable describing the total number of damaged sheets in 2.0; $N_1^{2.0} \sim$ Poiss ($\lambda_1^{2.0}$)
N - the total number describing the damaged sheets in 0.8 and 2.0 format is therefore:

$$N = N_1^{0.8} + N_1^{2.0} \tag{2}$$

The symbols $p_s^{0.8}$ and $p_s^{2.0}$ have been used to mark the probability of detecting a singular damaged sheet type 0.8 and 2.0 in the carrier`s processes.

$\left[(1-p_s^{0.8})N_1^{0.8} + (1-p_s^{2.0})N_1^{2.0}\right]$- the random variable describing the number of faulty sheets which have passed quality control, i.e. flaws generated by the supplier were not detected and the inspected sheets were considered free of flaws.

$\lfloor p_s^{0.8} N_1^{0.8} + p_s^{2.0} N_1^{2.0} \rfloor$- the random variable describing the amount of flawed sheets which have been detected and returned to the supplier,

$X_W = \max\{X^{0.8} - \lfloor p_s^{0.8} N_1^{0.8} \rfloor, 0\} + \max\{X^{2.0} - \lfloor p_s^{2.0} N_1^{2.0} \rfloor, 0\}$- the random variable describing the amount of sheets which have undergone quality control.

Marked by:

$y^{0.8}$ and $y^{2.0}$- the random variable describing the amount of forms cut out of a single steel sheet respectively for 0.8 and 2.0

In practice, the $y^{0.8}$ random variable, which is the number of templates from a 0.8 sheet, equals between 1 and 300 pieces. For 2.0 sheets the random variable assumes the value between 1 and 100 pieces. Let us note that variables $y^{0.8}$ and $y^{2.0}$ are the sum of indepentend Bernoulli tests with an identical probability of $p_y^{0.8}$ and $p_y^{2.0}$ respectively. Due to this observation and the analysis of the historical data it has been noted that the variables $y^{0.8}$ and $y^{2.0}$ may be modeled using the Pascal distribution of $y^{0.8}$ ~ Pascal ($r^{0.8}$, $p_y^{0.8}$ ) and $y^{2.0}$ ~ Pascal ($r^{2.0}$, $p_y^{2.0}$) respectively.

$$\forall k = 0, 1, 2, \ldots : P\left(y^{0.8} = k\right) = \binom{r^{0.8} + k - 1}{r^{0.8} - 1}\left(1 - p_y^{0.8}\right)^{r^{0.8}}\left(p_y^{0.8}\right)^k \tag{3}$$

$Y^{0.8}$ and $Y^{2.0}$- the random variables describing the total amount of forms produces from 0.8 and 2.0 sheets

Y - the random variable describing the total amount of forms produces from 0.8 and 2.0 sheets

$$Y = Y^{0.8} + Y^{2.0} = y^{0.8} \max\{X^{0.8} - \lfloor p_s^{0.8} N_1^{0.8} \rfloor, 0\} + y^{2.0} \max\{X^{2.0} - \lfloor p_s^{2.0} N_1^{2.0} \rfloor, 0\} \tag{4}$$

Let us note that $y^{0.8}$ (1- $p_s^{0.8}$) $N_1^{0.8}$ and $y^{2.0}$ (1- $p_s^{2.0}$) $N_1^{2.0}$ describes the total amount of forms produced from 0.8 and 2.0 damaged sheets which have not been detected and sent back to the supplier.

Furthermore $y^{0.8}$ ($X^{0.8}$ - $N_1^{0.8}$) and $y^{2.0}$ ($X^{2.0}$ - $N_1^{2.0}$) describes the total amount of forms produced from flawless 0.8 and 2.0 sheets. *Y* may therefore be recorded as formulas (5)

$$Y = y^{0.8}\left[\left(1 - p_s^{0.8}\right)N_1^{0.8}\right] + y^{0.8} \max\{X^{0.8} - N_1^{0.8}, 0\} + y^{2.0}\left[\left(1 - p_s^{2.0}\right)N_1^{2.0}\right] + y^{2.0} \max\{X^{2.0} - N_1^{2.0}, 0\} \tag{5}$$

Introducing a new random variable Z which describes the total amount of faulty forms as a result of all previous processes (at the provider and during the cutting process). We note that Z may be recorded as:

$$Z = Z_D^{0.8} + Z_Z^{0.8} + Z_D^{2.0} + Z_Z^{2.0} \tag{6}$$

where: $Z_D^{0.8}$ $Z_D^{2.0}$ - random variables describing the amount of faulty forms produced from good quality steel sheets sized 0.8 and 2.0 while $Z_z^{0.8}$ $Z_z^{2.0}$- random variables describing the amount of faulty forms produced from defective sheets which have not been detected, sized 0.8 and 2.0. Let us note that basing on the analyzed processes taking place in the company we may assume that:

$$Z_z^{0.8} \Big| \; y^{0.8}\left[\left(1 - p_S^{0.8}\right)N_1^{0.8}\right] \sim Poiss\left(\lambda_{Z_Z^{0.8}}\right) \text{ and } Z_D^{0.8} \Big| y^{0.8}\left(X^{0.8} - N_1^{0.8}\right) \sim Poiss\left(\lambda_{Z_D^{0.8}}\right)$$

$$Z_Z^{2.0} \Big| y^{2.0}\left[\left(1 - p_S^{2.0}\right)N_1^{2.0}\right] \sim Poiss\left(\lambda_{Z_Z^{2.0}}\right) \text{ and } Z_D^{2.0} \Big| \; y^{2.0}\left(X^{2.0} - N_1^{2.0}\right) \sim Poiss\left(\lambda_{Z_D^{2.0}}\right) \tag{7 - 10}$$

We expect that and since the amount of generated errors caused by external factors such as: erroneously chosen process parameters, inexperience of the operators etc. is similar for steel sheets bearing undetected defects as well as flawless ones. Furthermore, steel sheets bearing undetected defects increase the number of flawed forms in comparison to forms manufactured from flawless sheets. This is true even when assuming that a fraction of the defects generated in the cutting and bending processes includes the damaged areas of the sheet.

Knowing the layout of the random variable $Z_z^{0.8}$ | [$y^{0.8}$ (1- $p_s^{0.8}$) $N_1^{0.8}$] the layout of $Z_z^{0.8}$ may be deduced. The following calculations are carried out:

$$\forall k = 0, 1, 2, \ldots :$$

$$P\left(Z_Z^{0.8} = k\right) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} P\left(Z_Z^{0.8} = k \middle| \left[\left(1 - p_s^{0.8}\right)N_1^{0.8}\right] = l \wedge y^{0.8} = m\right) \cdot P\left(\left[\left(1 - p_S^{0.8}\right)N_1^{0.8}\right] = l \wedge y^{0.8} = m\right) =$$

$$= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} P\left(Z_Z^{0.8} = k \middle| \left[\left(1 - p_S^{0.8}\right)N_1^{0.8}\right] = l \wedge y^{0.8} = m\right) \cdot P\left(\left[\left(1 - p_S^{0.8}\right)N_1^{0.8}\right]\right) \cdot P\left(y^{0.8} = m\right) \tag{11}$$

The second eqiality results from the fact that the random variables [(1- $p_s^{0.8}$) $N_1^{0.8}$] and $y^{0.8}$ are independent.

Similar calculations have been carried out for the $Z_D^{0.8}$ ; $Z_z^{2.0}$ ; $Z_D^{2.0}$ variables

After the cutting process is complete there are four groups of damaged forms. Let us introduce the following markings describing the volume of forms not being admitted to the edging process.

$p_D^{0.8}$ - percentage of detected forms from faulty forms manufactured from flawless sheets 0.8.

$p_Z^{0.8}$ - percentage of detected forms from faulty forms manufactured from damaged sheets 0.8.

$p_D^{2.0}$ - percentage of detected forms from faulty forms manufactured from flawless sheets 2.0.

$p_Z^{2.0}$ - percentage of detected forms from faulty forms manufactured from damaged sheets 2.0.

*W* has been marked as the random variable describing the total amount of forms being admitted to the edging process. *W* may therefore be recorded in the following form (12):

$$W = Y - \left[p_D^{0.8} Z_D^{0.8}\right] - \left[p_Z^{0.8} Z_Z^{0.8}\right] - \left[p_D^{2.0} Z_D^{2.0}\right] - \left[p_Z^{2.0} Z_Z^{2.0}\right] \tag{12}$$

Let us now introduce the $W_D^{0.8}$ | [(1 - $p_D^{0.8}$) $Z_D^{0.8}$] and $W_Z^{0.8}$ | [(1 - $p_Z^{0.8}$) $Z_Z^{0.8}$] random variables describing the amount of faulty forms generated in the processes thus far [6]. We consider the bending process for flawless and faulty 0.8 sheets while assuming that we obtain [(1 - $p_D^{0.8}$) $Z_D^{0.8}$] and [(1 - $p_Z^{0.8}$) $Z_Z^{0.8}$] faulty undetected forms from the cutting process on both flawless and damaged steel sheets. Analogically defining random variables $W_D^{2.0}$ | [(1 - $p_D^{2.0}$) $Z_D^{2.0}$] and $W_Z^{2.0}$ | [(1 - $p_Z^{2.0}$) $Z_Z^{2.0}$] on 2.0 sheets. These variables are identifiable, therefore it may be assumed that:

$$W_D^{0.8} \mid \left[\left(1 - p_D^{0.8}\right)Z_D^{0.8}\right] \sim Poiss\left(\lambda_{W_D^{0.8}}\right) \quad \text{and} \quad W_D^{2.0} \mid \left[\left(1 - p_D^{2.0}\right)Z_D^{2.0}\right] \sim Poiss\left(\lambda_{W_D^{2.0}}\right)$$

$$W_Z^{0.8} \mid \left[\left(1 - p_Z^{0.8}\right)Z_Z^{0.8}\right] \sim Poiss\left(\lambda_{W_Z^{0.8}}\right) \quad \text{and} \quad W_Z^{2.0} \mid \left[\left(1 - p_Z^{2.0}\right)Z_Z^{2.0}\right] \sim Poiss\left(\lambda_{W_Z^{2.0}}\right) \tag{13 - 16}$$

Among the errors generated during the bending process there are ones which resulted from erroneously choosing the parameters, such as the operator`s errors and occasional technical errors. By knowing the layout of the conditional distributions, layouts of the $W_D^{0.8}$, $W_Z^{0.8}$, $W_D^{2.0}$ and $W_Z^{2.0}$ variables may be deduced.

$$\left(\lambda_{Z_Z^{0.8}}\right) \geq \left(\lambda_{Z_D^{0.8}}\right) \; \left(\lambda_{Z_Z^{2.0}}\right) \geq \left(\lambda_{Z_D^{2.0}}\right)$$

$$\forall k = 0, 1, 2, \ldots : P\left(W_D^{0.8} = k\right) = \sum_{l=0}^{\infty} P\left(W_D^{0.8} = k \middle| \left[\left(1 - p_D^{0.8}\right)Z_D^{0.8}\right] = l\right) \cdot P\left(\left[\left(1 - p_D^{0.8}\right)Z_D^{0.8}\right] = l\right) \tag{17}$$

The remaining distributions may be defined analogically. Let $W_B$ = $W_D^{0.8}$ +$W_Z^{0.8}$ +$W_D^{2.0}$ +$W_Z^{2.0}$ be the random variable describing the total number of damaged forms produced in the entire manufacturing process. Let us calculate expected value of the $W_B$ variable. Note that $E(W_B) = E(W_D^{0.8})+E(W_Z^{0.8})+E(W_D^{2.0})+E(W_Z^{2.0})$. Individual expected values may be calculated by utilizing the following formula:

$$E\left(W_D^{0.8}\right)=\sum_{k=0}^{\infty}k\cdot P\left(W_D^{0.8}=k\right)=\sum_{k=0}^{\infty}k\cdot\sum_{l=0}^{\infty}P\left(W_D^{0.8}=k\left|\left[\left(1-p_D^{0.8}\right)Z_D^{0.8}\right]=l\right.\right)\cdot P\left(\left[\left(1-p_D^{0.8}\right)Z_D^{0.8}\right]=l\right) \tag{18}$$

The total expected amount of faulty forms T including the detected and discarded ones may now be logged as (19):

$$E(T)=E(W_B)+E\left(\left[p_D^{0.8}Z_D^{0.8}\right]\right)+E\left(\left[p_Z^{0.8}Z_Z^{0.8}\right]\right)+E\left(\left[p_D^{2.0}Z_D^{2.0}\right]\right)+E\left(\left[p_Z^{2.0}Z_Z^{2.0}\right]\right) \tag{19}$$

## 3. CONCLUSION

By utilizing the presented model it is possible to estimate the expected amount of produced sheets not matching quality demands. After assigning the costs, we may also assess the value of errors depending on the stage they occur in. It is also crucial to be aware of the costs spent on producing forms from faulty sheets which have not been discarded during initial quality control. The model allows estimating the amount of forms not matching quality demands depending on the quality of the delivered steel sheets and the quality of the cutting and edging processes. The errors generated on chosen 0.8 and 2.0 sheets were not codependent. During analysis, reliance between errors in sheets of the same format has been noted, therefore future studies will consider a model assuming errors between sheets varying solely in width and length.

Being able to assess the amount of errors generated on individual manufacturing levels allows optimizing the supply level and production costs for various stages of the production process. Additionally the amount of waste may be foreseen for each stage, which facilitates control of the process and production time. Precisely estimating the respective parameters of the analyzed distributions basing on the gathered data allows for an efficient forecast of error trends.

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