A SIMPLIFIED CALCULATION OF SOME VIEW FACTORS FOR THE CRUCIBLE FURNACE

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Abstract

For the melting of mostly non-ferrous metals or for keeping metals in the liquid state crucible furnaces are often used. Greater use of crucible furnaces for steel and cast iron prevents the relatively high energy intensity of these furnaces. To reduce the energy intensity could contribute even a mathematical optimization of the parameters of the cup. Part of the mathematical model of the radiation heat transfer is the calculation of the view factors. The view factor is mathematically a surface integral whose integrand is again a surface integral. The analytical solution of this problem is therefore possible only in special cases. This article deals with one of these cases and includes a proposal to simplify the calculation of some of the local view factors in the crucible furnace. The proposed calculation is based on the fact that the outer surface of the crucible and the inner surface of crucible furnaces are rotational surfaces with a common axis.

Keywords: Steel and cast iron, non-ferrous metals**,** crucible *furnace*, view factor

1. INTRODUCTION

Crucible furnaces designed for melting, as well as for maintaining the melt in the liquid state, are demanding energy consumption considerably. In addition to many other factors, the shape of the crucible can also influence energy consumption. Some of the challenging experiments can be replaced using a mathematical model in assessing various shapes of the crucible or for designing a new one. The mathematical models in which numerical calculations of various dependencies are replaced by the analytical expression of these dependencies are more preferable for the examination. This article deals with a part of the mathematical problem of calculating radiation heat transfer in the crucible furnace. In the article there is proposed a simplified analytical expression of the local view factor for the certain shape of the crucible.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The calculation of the radiation heat transfer between the crucible and the furnace, i.e. between the crucible (body *A* bounded by surface S_A) and the furnace (body *B* with the inner surface S_B) can be mathematically expressed by the surface integral over the surface S_A whose integrand is a surface integral over the surface S_B (see [1], [2], [3]). This article deals only with usual case characterized by two conditions:

The crucible (solid of revolution) is centrally located in a cylindrical furnace, i.e. surfaces S_A and S_B are rotational and coaxial.

All the points of the surface S_A of the crucible at a certain height have the same temperature and the temperature of all points of the area S_B depends only on their height (measured from the bottom of the crucible).

Given these assumptions, the knowledge of the view factors of element dS_A to the whole wall of the furnace (respectively to their belts between the heights h_1 and h_2) and the bottom and the lid (respectively to their any annuluses) is sufficient for practical use.

The basis of calculating over the surface S_B is the so-called *local view factor*. Local view factor is a dimensionless number that expresses what proportional part of the radiation from the appropriate dS_A element of the surface S_A falls on a certain part of surface S_B (the rest of the radiation falls on another part of the surface S_B or falls on the surface S_A itself, if it is concave).

The local view factor (in the following referred to as the view factor) for radiation from the dS_A element to a certain part of surface S_B (**Figure 1**) is given by

$$\Phi_{A,B} = \frac{1}{\pi} \int_{S_{B^*}} \frac{\cos\alpha_A \cdot \cos\alpha_B}{d^2} \cdot dS_B \ , \qquad (1)$$

where dS_B is the element of surface S_B. Angles α_A, α_B are the angles between the normals of elements dS_A, dS_B and the line connecting both elementary surfaces; d is the distance between dS_A and dS_B. Furnace interior is the surface of the cylinder that, for purposes of this article, will be divided only into three sub-areas: bottom, wall and lid. For the calculation of the integral the whole surface S_B is not taken in account, but only the "visible" part of the S_B marked as S_{B^*}. The dS_A element radiates only into the outer half-space.

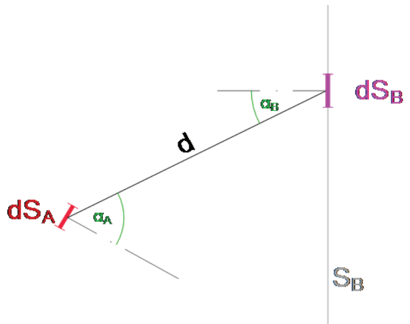

Figure 1 Local view factor for radiation from the dS_A element to the dS_B element of the surface S_B

3. PROPOSED SOLUTION FOR THE LOCAL VIEW FACTOR

In the available resources [4] only one analytical solution of formulated problem has been found. This solution is directly applicable only in the case when the area S_A of the crucible is cylindrical. The formula in [4] expresses the view factor of dS_A element on the bottom, respectively on the lid of the furnace. Since the sum of the view factors of dS_A element on the bottom, on the wall and on the lid of the furnace is equal to 1, the view factor of dS_A element on the wall of the furnace can be easily calculated. The solution in [4] is therefore considered to a crucible different from the cylindrical shape directly applicable just for the vertical elements dS_A, which are on the "widest" cylindrical part of the crucible.

In the following text a cylindrical coordinate system determined by the axis of the furnace and the bottom of the furnace (furnace bottom will have coordinate 0) will be used together with marking:

R_B - the radius of the furnace,
R_A - the radius of the crucible,
H - the height of the visible part of the furnace wall (see **Figure 2**),
r_A, h_A - the coordinates of the dS_A element (radius, height), under given simplifications $r_A = R_A$.

The proposed simplification of the calculation of the view factor for the vertical dS_A element is based on the fact that the surfaces S_A and S_B are both rotational and coaxial.

The **Figure 2** shows the horizontal cut through the areas S_A and S_B, where the solid green line represents two dS_{A1} and dS_{A2} elements which are of the same size and which are at the identical height. Each of the elements determines the elementary cylindrical sector which corresponds to the area of same size on the furnace wall (in **Figure 2** appears as an orange arch). It is therefore obvious, that the same amount of radiation, which one element dS_{A1} emits to a certain height of the sector of the dS_{A2} element, exudes the dS_{A2} element to the same height of the sector of the element dS_{A1}.

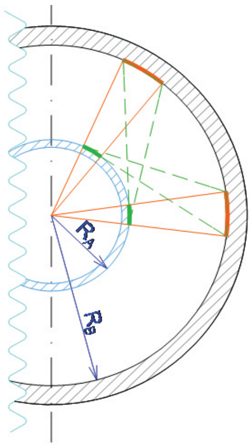

Figure 2 Horizontal cut through the crucible and the furnace

This fact has been already used in [5] where the required view factors of the dS_A element has been suggested to replace view factors obtained under fictitious assumption that each element emits everything just in its own sector.

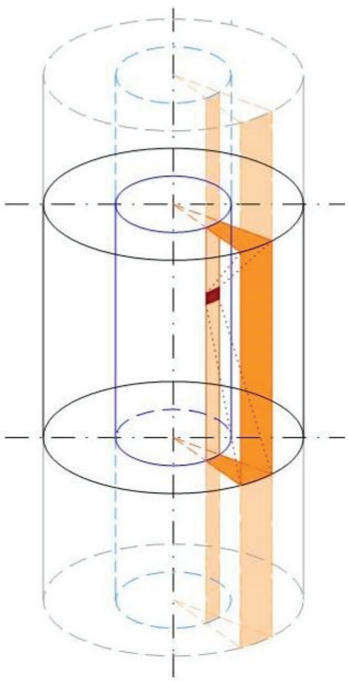

Figure 3 The extension of the wall of the furnace

Using the same simplification for the bottom and the lid of the furnace made in some cases a larger deviation from the results according to [4], particularly in case that the dS_A element was near the bottom or lid of the furnace.

These issues, however, managed to remove the consistent application of original idea when used for the infinitely long cylindrical surface, which is an extension of the cylindrical surface of the inner wall of the furnace (**Figure 3**).

The **Figure 3** shows characteristic areas of the crucible and the furnace drawn as transparent, considered extension to the interval $(-\infty, +\infty)$ is indicated by lighter lines.

For this infinite cylindrical surface the ***basic hypothesis*** has been pronounced: **Each dS_A element is getting back to the all heights of their sector almost same amount as it radiates into other sectors.**

The basis of the calculation of the required view factors in the proposed solution is therefore the integral for the calculation of the amount of radiation on the part of the lateral surface of the (elemental) sector infinitely long cylindrical surface between the heights h_1 and h_2.

For the calculation of the view factors, it is advantageous to standardize this integral so that its value on the interval $(-\infty, +\infty)$ has been equal to 1 (by definition is 1 required value for the sum of the view factors of the dS_A element on all visible surfaces).

The expression of the integral greatly facilitates the fact that the angles α_A and α_B of (1) under given conditions fulfil

$$\alpha_A = \alpha_B \quad . \tag{2}$$

The approximate expression of the view factor *I(h1,h2)* of the dS_A element on the visible part of the infinite cylindrical surface between the heights h_1 and h_2 under these conditions has the form

$$I(h_1,h_2) = \frac{2}{\pi} \cdot \int_{h_1}^{h_2} \frac{(R_B - r_A)^3}{\left((R_B - r_A)^2 + (h - h_A)^2\right)^2} \cdot dh \quad . \tag{3}$$

View factors of dS_A element on the bottom, wall and lid of the furnace then have the values $I(-\infty, 0)$, $I(0, H)$ and $I(H, \infty)$, where H is the height of the furnace.

The amount of radiation that would correspond to the part of the infinite cylindrical surface under the bottom of the furnace, is used to calculate the view factor of the bottom of the furnace, for example. Similar advance can be used for the lid of the furnace.

Primitive function of the integral (3) can be analytically expressed in the form

$$\frac{1}{\pi} \cdot \left(\arctan\left(\frac{h - h_A}{R_B - r_A} \right) + \frac{(R_B - r_A)(h - h_A)}{(R_B - r_A)^2 + (h - h_A)^2} \right) \quad . \tag{4}$$

By formulas (3), respectively (4), not only view factors for the bottom, wall and lid of the furnace can be calculated, but also the view factors for an arbitrary band of the furnace wall which is determined by heights h_1 and h_2, and for any annulus of the bottom or the lid of the furnace which is determined by the radii r_1 and r_2. Calculating the view factor for the annulus of the bottom or the lid of the furnace it is necessary in the first to assign heights h_1 and h_2 to radii r_1 and r_2.

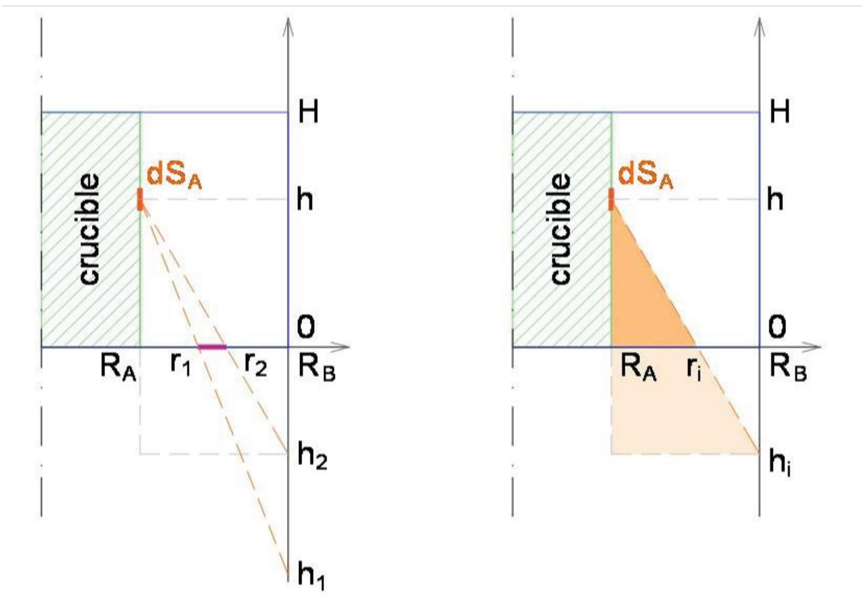

Figure 4 $h_i = f(r_i)$

The height h_i which corresponds to a certain radius r_i at the bottom or at the lid of the furnace is obtained by projection of the radius r_i on the endless cylindrical surface (for the bottom of the furnace, see **Figure 4**).

From the similarity of triangles in **Figure 4** for the radius r_i from the bottom of the furnace the relationship follows:

$$h_i = h - \frac{(R_B - r_A)h}{r_i - r_A} \quad . \tag{5}$$

Similarly, for the radius r_i of the furnace lid holds:

$$h_i = h + \frac{(R_B - r_A)(H - h)}{r_i - r_A} \quad . \tag{6}$$

The accuracy of the proposed method (simplified calculation of the view factors) was tested using [4] .

It was done a greater amount of the calculations of the view factors of the dS_A element on the wall of the furnace for different input parameters.

The results obtained by the proposed method differed not more than 5% from the results according to [4], the vast majority of the results, however, differed by less than 2%. For most practical calculations the proposed method can be considered as sufficiently accurate.

The simplicity of calculations is not the greatest benefit of the proposed method, but the fact that using the same principle can be similarly expressed the view factors for dS_A element which is not vertical. Validation of the obtained results, however, is challenging and has not been completed yet.

4. CONCLUSION

The proposed method of the calculation of the local view factor for the crucible furnace can be easily used for cylindrical crucible by the calculation of radiation heat transfer not only in case where the furnace temperature is constant, but also where the wall temperature dependents only on the distance from the bottom of the furnace and the temperature of the bottom and the lid of the furnace dependents only on the distance from the axis of the furnace.

To the further development of the mathematical modeling of the radiation heat transfer between the crucible and the furnace would have greatly helped the ability to calculate the view factors also for the more general shape of the crucible. Relevant proposals obtained by using listed here the basic hypothesis, however, has not been sufficiently verified yet.

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