

## ESTIMATING OF THE SHAPE AND SIZE THE LIQUID AND MIXED ZONE IN THE MODELLING OF PROCESSES WITH SEMI-SOLID CORE

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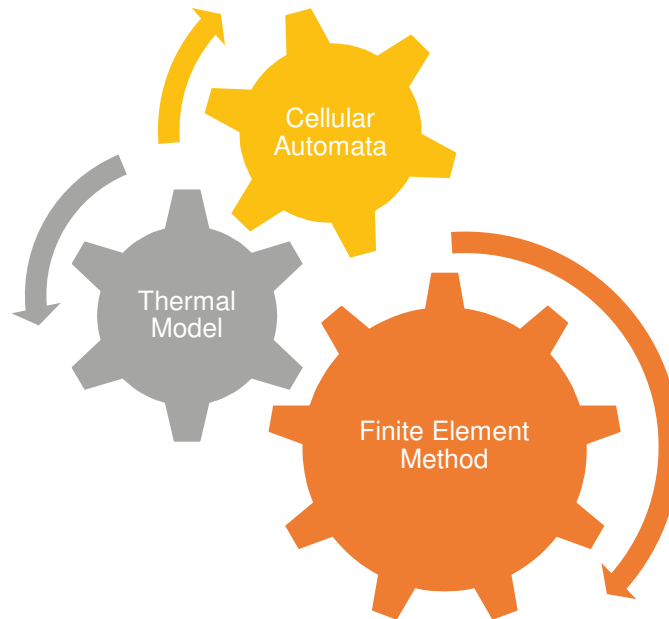
### Abstract

The article presents the use of multiscale model to estimate the shape and size of the mixed and liquid zone in the modelling processes with semi-solid core. On a macro level to calculate the heat flow is used three-dimensional thermal model based on the Finite Element Method. While at the micro level it was used a Cellular Automaton (CA) combined with the finite element method based on the work of Gandin and Rappaz. Furthermore, the state of the cells influences to the temperature distribution it can be concluded that there is a bidirectional coupling. Each cell changes its state from liquid through mixed to the solid. Thanks to the use of cellular automata it can be possible both a better understanding of the process of creation and estimation the mixed and liquid zone. The paper presents the proof of concept of the original application allows to estimate and visualize the mixed and liquid zone.

**Keywords:** Semi-solid state, finite elements method, cellular automaton, physical simulation

### 1. INTRODUCTION

The problem of modelling the rolling of flat bands of semi solid core requires the essence of the process from the mechanical and heat perspective. Existing technologies, such as casting sheet between two rolls, for several years, have used information occurring the same phenomena obtained by empirical methods. Cooling the strands after continuous casting process in a conventional manner and the re-heating it for the plastic working is associated with considerable loss of energy. There is evidence justifying the need to build models describing rolling bands with semi-solid (mixed) zone in the middle. Creating a mathematical model describing the physical phenomena occurring during rolling bands of semi-solid core will allow us to predict changes in the parameters of the process, ensure proper design of rolling technology and reduce significantly the number of industrial trials. It will be possible to control the rolling process and reduce the production costs. In the strip casting process steel is cast in to a thin gage, reheated and rolled into final production a limited number of passes. This is due to elimination of reheating to the very high temperature of rolling which significantly decreases the level of yield stress. The new technologies are characterized by drastic reduction of the manufacturing costs (low level of energy and water consumption, as well as CO<sub>2</sub> emission), high productivity and capacity and low investment costs. For these reasons it is very important to calculate the zone with the mixed phases. This zone has very irregular and complex shape. That's why mathematical model should be a spatial model. In the case of a good description of the mixed zone would be possible to better control the rolling process. It is very difficult because model should works in two levels of calculation: macro and micro. The thermal model used in this work determines the temperature in the nodes of the mesh at the macro level. However, it is not sufficient to calculate the contribution of the liquid and solid phase in the core of slab. Therefore it introduced an additional module for calculating the participation of these phases. It is a model of cellular automata working on a micro scale. The mathematical model may also be useful in conventional continuous casting systems especially in final stage of rolling to improve the shape and mechanical properties of slabs. For this purpose has been developed hybrid model (**Figure 1**). Finite element method was used to calculate the temperature distribution. The calculated temperature is the basis for cellular automata allows determining the shape of the mixed zone.



**Figure 1** Parts of the hybrid model

## 2. THERMAL MODEL

In the case of metal forming processes, we have to deal with non-stationary heat flow. In addition, the process is complicated and, therefore, uses three-dimensional models. Therefore, for a given point in time derivatives of the temperatures they are considered as functions of the coordinates  $(x, y, z)$ . In the presented model it was based on the general diffusion equation for a three-dimensional thermal problem, which is called Fourier-Kirchhoff equation, written in a form [1]:

$$\frac{\partial}{\partial x} \left( \lambda_{xx} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda_{yy} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda_{zz} \frac{\partial T}{\partial z} \right) + (Q - \rho c_p \frac{\partial T}{\partial t}) = 0 \quad (1)$$

In (1)  $T$  is the temperature distribution [K],  $\lambda$  is the heat conduction coefficient [ $W/(m \cdot K)$ ],  $Q$  is the rate of heat generation due to plastic deformation or current intensity flow (heating process) [ $W/m^3$ ],  $c_p$  denotes the specific heat [ $J/(kg \cdot K)$ ] and  $\rho$  is the density [ $kg/m^3$ ].

As a result, we obtain a matrix notation solution heat conduction equation for the case of unsteady state:

$$HT + C \frac{\partial}{\partial t} T + P = 0 \quad (2)$$

where:  $H$  is the thermal conductivity matrix,  $C$  is a heat capacity matrix,  $T$  is the nodal temperature vector and  $P$  is the vector of free terms. The heat capacity matrix was calculated from equation (0).

$$C = \int_V \rho c_p NN^T dV \quad (3)$$

In the general case the temperature  $T$  of the nodes depends on the time [s]. Assuming that the vector is a vector  $T_0$  nodal temperature at time  $t = 0$ , the period of time  $\Delta t$  vector is given by the equation:

$$T = \{N_0, N_1\} T_0 \quad (4)$$

where:  $N_0, N_1$  means the shape functions dependent on time.

Assuming that the temperature dependence of small intervals of time  $\Delta t$  is the nodal linear shape functions take the form:

$$N_0 = \frac{\Delta t - t}{\Delta t} \text{ and } N_1 = \frac{t}{\Delta t} \quad (5)$$

Taking into account the relation (5), derivatives of temperature versus time can be written as follows:

$$\frac{\partial T}{\partial t} = \left\{ \frac{\partial N_0}{\partial t}, \frac{\partial N_1}{\partial t} \right\} \begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} = \frac{1}{\Delta t} \{-1, 1\} \begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} \quad (6)$$

Because the vector of nodal temperature  $T_0$  is known, than carry out the integration of a matrix equation for the steady state with respect to time should enter one residual weighted according to:

$$\int_0^{\Delta t} \frac{t}{\Delta t} \left[ H \{N_0, N_1\} \begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} + C \left\{ \frac{\partial N_0}{\partial t}, \frac{\partial N_1}{\partial t} \right\} \begin{Bmatrix} T_0 \\ T_1 \end{Bmatrix} + P \right] dt = 0 \quad (7)$$

By introducing a time dependent shape functions (5) into expression (7) we obtain:

$$\int_0^{\Delta t} \frac{t}{\Delta t} \left[ H \left( \frac{\Delta t - t}{\Delta t} T_0 + \frac{t}{\Delta t} T_1 \right) + \frac{C}{\Delta t} (T_1 - T_0) + P \right] dt = 0 \quad (8)$$

As a result, we obtain a matrix notation solution heat conduction equation for the case of unsteady state:

$$\left( 2H + \frac{3}{\Delta t} C \right) T_1 + \left( H - \frac{3}{\Delta t} C \right) T_0 + 3P = 0 \quad (9)$$

Equation (9) is an algebraic equation, which allows the calculation of the nodal temperatures  $T_1$  at the time  $\Delta t$  when the selected temperature  $T_0$  at  $t = 0$ . Equation (9) is derived by assuming a linear relationship between the temperature in the interval of time  $\Delta t$ . The applied solution was used Galerkin integration scheme, due to the fact that it gives good results for the boundary conditions existing in the rolling.

For the rolling process in order to track changes in temperature, mixed boundary conditions applied, consisting of a second and third boundary condition:  $q_b = g(t)$ ,  $\lambda \frac{\partial T}{\partial n} + \alpha(T - T_0) = 0$ . In these expressions,  $n$  is a vector normal to the surface,  $\alpha$  is the heat transfer coefficient [ $W/(m^2 \cdot K)$ ] and  $T_0$  is the temperature of the surrounding medium. In addition on the end of the rolled band, was used a first boundary condition  $T_b = f(t)$ .

### 3. CELLULAR AUTOMATON MODEL

Another part of the hybrid model operating in the micro area is a cellular automaton model allow the estimation of the state of individual cells. It is allowing to calculate the mixed zone. Cellular automata were inserted into the mesh of finite elements. In each element of mesh is inserted 100 of cellular automata in each of the directions of the coordinate system. Temperatures for the initial cellular automata based on the interpolation of the temperature from the nodes of elements from FE mesh obtained from the thermal model. Then, using the model of Gandin and Rappaz it has been calculated the new temperatures, which were transferred to the FE mesh nodes. Each cell can have three states from the range (0; 1), respectively, 0 - solid phase: 1 - liquid phase (0, 1) - a mixed zone. Cellular automaton is working in the loop in micro level. This model recalculating the temperatures and frictions of solid of the FE nodes after each micro time step.

The variation of the volume fraction of solid of a cell  $v - \delta f_{s,v}$  - is computed during one micro-time-step following the scheme proposed by Gandin and Rapazz [2, 3]. There are three different possibilities:

- a cell is completely solidified at the beginning of the micro-time-step. It is the situation when temperature is below the solidus or stays liquid during the micro-time-step. In both cases  $\delta f_{s,v} = 0$
- a cell becoming mushy. Due to nucleation the state index of a liquid cell changes to a nonzero value during the micro-time-step. The increment of solid fraction is initialized using the Scheil micro segregation model at the actual temperature of the cell  $T_v^t$ ,  $\delta f_{s,v} = \frac{T_v^t - T_s}{T_L - T_s}$ , where  $T_L$  is the liquidus of the alloy,  $T_s$  is the melting point of the solvent.

The enthalpy variation for each node  $n$  is given by equation:

$$\delta H_n = \rho c_p [T_n^{t+\delta t} - T_n^t] - \Delta H_f \delta f_{s,n} \quad (10)$$

Dividing both sides of the equation by  $\rho c_p$  obtain the following equation

$$\frac{\delta H_n}{\rho c_p} = [T_n^{t+\delta t} - T_n^t] - \frac{\Delta H_f \partial f_{s,n}}{\rho c_p} \quad (11)$$

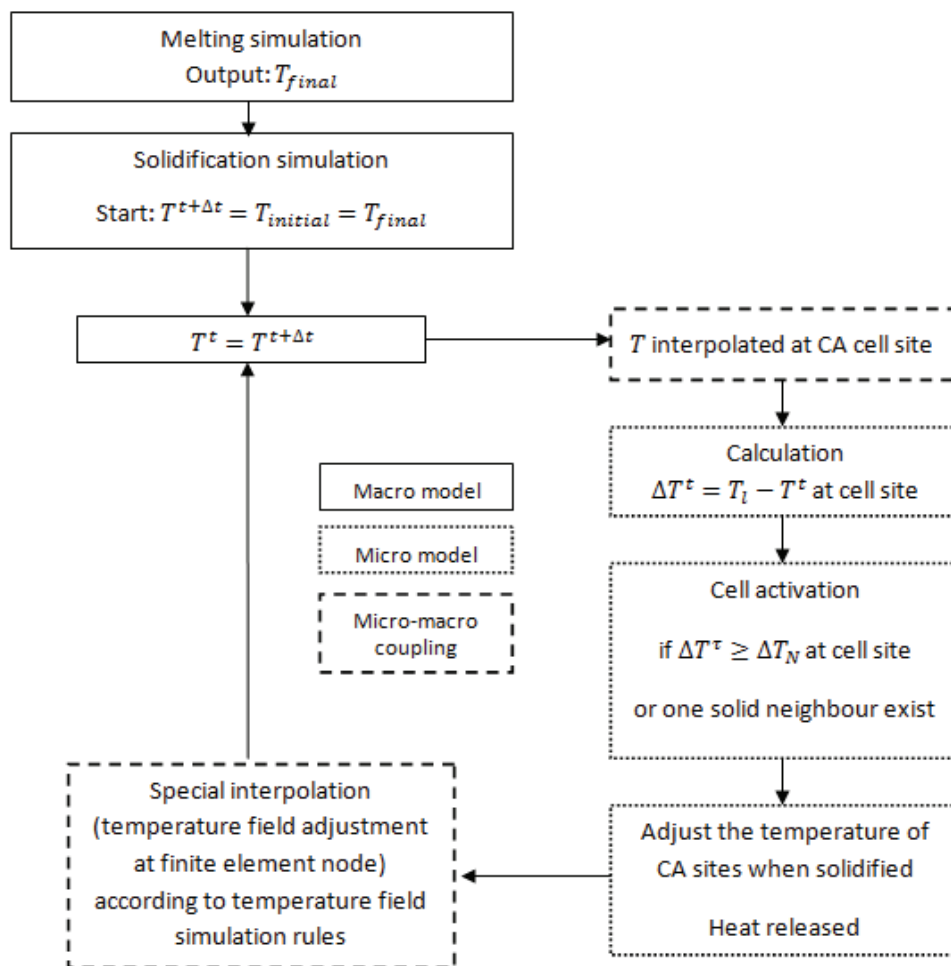
After ordering receive and the new temperatures at the nodal point can be calculated according to:

$$T_n^{t+\delta t} = T_n^t + \frac{\delta H_n + \Delta H_f \partial f_{s,n}}{\rho c_p} \quad (12)$$

The next step of heat flow calculation in macro level (thermal model based on FE) can be carried out using these updated temperatures.

#### 4. MULTISCALE MODEL OF MIXED ZONE ESTIMATE

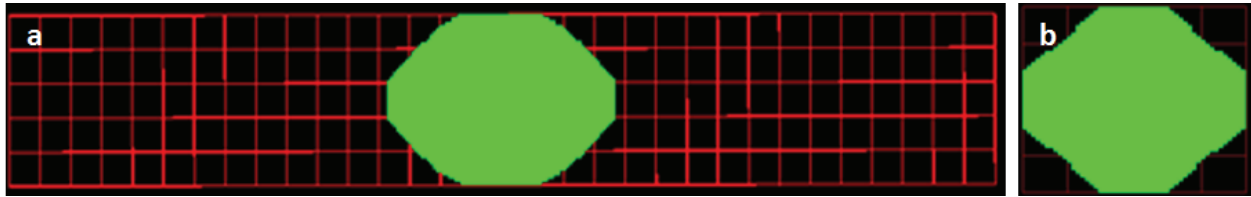
To estimate the mixed zone was created a hybrid model based on the finite element method and uses in each time step cellular automata [4]. By using cellular automata it is possible to identify components of the mixing zone from the liquid phase through mixed zone, to the solid phase. Algorithm for calculation fraction of a liquid phase is shown in **Figure 1**.



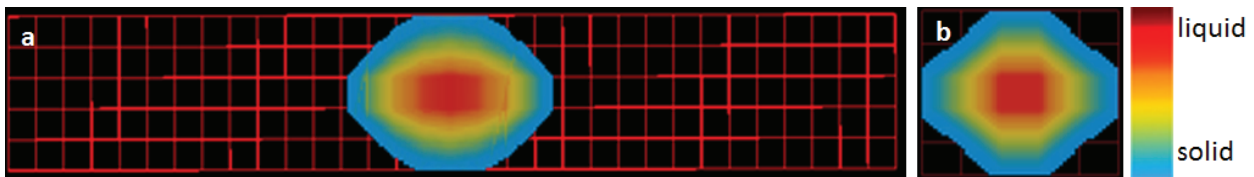
**Figure 2** The hybrid algorithm for mixed zone calculation

The described algorithm (**Figure 2**) has been implemented, and thus created a proof of concept application that allows for the determination of mixed zone. The developed application allows you to visualize the results of calculations of the mixed zone. It is possible to present the results in longitudinal section and transverse

(respectively labelled as a and b) both the shape of the mixing zone (**Figure 3**) as well as participation phase (**Figure 4**).



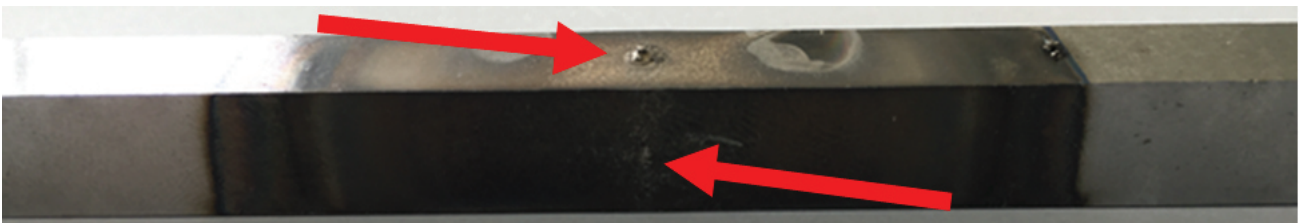
**Figure 3** The shape of the mixed zone: a) longitudinal section, b) cross section



**Figure 4** The liquid fraction distribution: a) longitudinal section, b) cross section

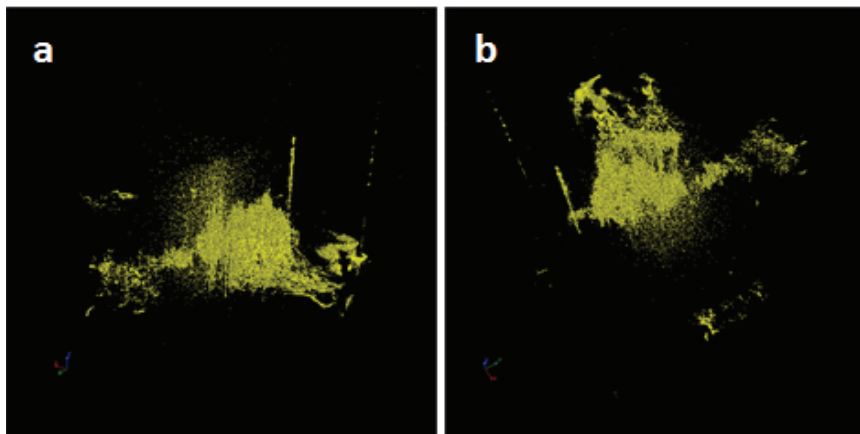
## 5. THE EXPERIMENTAL RESEARCH AND THE MATHEMATICAL MODEL VERIFICATION

Experimental tests were performed at the Institute of Ferrous Metallurgy in Gliwice. The dimensions of the samples of the steel used in the studies are 10 x 10 x 125 mm. During the experiment samples were heating to a temperature approaching the solidus and then cooled. After the experiment, on the sample surface can be observed characteristic points in the centre of each surface. These points are shown by the arrows on **Figure 5**. Those points appeared due to melting of the material [4].



**Figure 5** View of the sample after experiment

Executed also computed tomography (CT) described above steel samples. While research has been shown the porous areas after the solidification process. The porous zone (**Figure 6**) was created perpendicular to each of sample surface as show on **Figure 5**.



**Figure 6** Results from the computer tomography

## 6. CONCLUSIONS

The results obtained with the developed program based on the created hybrid model are consistent with tests conducted on real samples. In addition, the correctness of the obtained results confirms the results obtained with CT. This provides the basis for this, to develop a discussed model. Using of that type of models allows to savings in the rolling process. The developed application is the original application allows estimating and visualizing the mixed and liquid zone. In that the analysis is faster and better.

The visualization process is very time consuming because of the large number of cells in the CA algorithm. Therefore, it is considering the use of parallel algorithms [5]. This should accelerate the program.

## ACKNOWLEDGEMENTS

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