

**PREDICTION MODEL OF THE FLOW STRESS FOR THE COMPUTER-AIDED DESIGN HOT ROLLING SHEET AND STRIPS PATTERN**

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**Abstract**

A model of prediction of flow stress during hot rolling plates and wide strips of carbon, low alloy and micro-alloyed steels, especially deformation resistance which are not yet known, was developed. Method of L. V. Andreyuk was adopted for a basis of model, wherein each of thermomechanical factors represented as equations for the 13 chemical elements. The main limitations of the method (rolling temperature not less 800 °C, strain 0.05-0.3) are overcome by supplementing the known dependence factor, which takes into account the phase transformation, as well as a factor that compensates for the degree of error in the calculation and strain rate. These factors have reduced the lower limit of the confidence interval for the temperature to 700 °C and raise the upper limit of the confidence interval for the degree of deformation up to 0.5. The degree of conformity predicted and observed values of the flow stress is 82.6 - 94.7%.

**Keywords:** Steel with an arbitrary chemical composition, hot rolling sheet, flow stress, thermomechanical factors, factor of influence the phase transformations, correction factor by the strain and strain rate

**1. INTRODUCTION**

In the development and analysis of modes of rolling it is necessary to perform constraint checking of the force, torque and power rolling. For the projection of said parameters it is necessary to calculate the resistance deformation of the metal, which is associated with a yield stress of the metal under these thermomechanical factors (strain, strain rate and temperature of deformation). In Russia used the term "True Yield Stress" [1]. Abroad used the terms "Mean Flow Stress" and "Flow Stress" [2].

In order to display the inconstancy of strength characteristics of the rolled metal due to variations of melting chemical composition, as well as for the development of the rolling steel grades, features of resistance deformation are not yet known, a special software of computer-aided design must contain the rolling forecasting models yield strength steel with an arbitrary chemical composition. For the computer-aided design of cold rolling a similar model is presented in our paper [3]. Model for prediction Flow Stress of the hot rolling is based on method developed by Andreyuk.

**1.1. Features method of Andreyuk**

To calculate the Flow Stress, Andreyuk et others [4] suggested dependence, built on the results of tests on the plastometer according to the method of thermomechanical factors:

$$\sigma_F = \sigma_{F0} \cdot \underbrace{\vartheta^a}_{K_\vartheta} \cdot \underbrace{(10 \cdot \xi)^b}_{K_\xi} \cdot \underbrace{(\theta/1000)^c}_{K_\theta} \tag{1}$$

where  $\sigma_{F0}$  - Flow Stress at "standard" conditions of the test;  $K_\xi$ ,  $K_\vartheta$  и  $K_\theta$  - factors of influence of strain ( $\xi$ ), strain rate ( $\vartheta$ ) and temperature of deformation ( $\theta$ );  $a, b, c$  - empirical factors. The values  $\sigma_{F0}$ ,  $a, b$  and  $c$  are calculated based on the chemical composition of steel:

$$\sigma_0 = 66.8 + 0.1 \left[ (k'_1 X_1 + k''_1 X_1^{1.5}) + \dots + (k'_{13} X_{13} + k''_{13} X_{13}^{1.5}) \right]; \quad (2)$$

$$a = 0.126 + 0.01 \left[ (l'_1 X_1 + l''_1 X_1^{1.5}) + \dots + (l'_{13} X_{13} + l''_{13} X_{13}^{1.5}) \right]; \quad (3)$$

$$b = 0.125 + 0.01 \left[ (m'_1 X_1 + m''_1 X_1^{1.5}) + \dots + (m'_{13} X_{13} + m''_{13} X_{13}^{1.5}) \right]; \quad (4)$$

$$c = -2.82 + 0.01 \left[ (n'_1 X_1 + n''_1 X_1^{1.5}) + \dots + (n'_{13} X_{13} + n''_{13} X_{13}^{1.5}) \right]; \quad (5)$$

where  $X_1, \dots, X_{13}$  - symbols of chemical elements;  $k', k'', l', l'', m', m'', n', n''$  - factors obtained from the results of plastometry studies (**Table 1**).

**Table 1** Symbols of chemical elements and factors for the calculation of the Flow Stress [4]

Chemical element	Symbol	Factor							
		$k'$	$k''$	$l'$	$l''$	$m'$	$m''$	$n'$	$n''$
C	$X_1$	-65.7	141	9.17	-5.24	23.0	-18.6	-63.0	43.1
Mn	$X_2$	134	-36.2	-0.314	0.107	2.37	-0.591	-25.6	8.07
Si	$X_3$	31.9	-37.8	-4.98	3.57	5.30	-3.39	59.3	-45.5
Cr	$X_4$	155	-31.3	-0.29	0.0612	1.32	-0.358	-15.9	2.66
Ni	$X_5$	70.6	-5.04	-0.315	0.0319	0.450	-0.037	7.28	-0.633
W	$X_6$	-155	40.1	0.559	-0.148	1.90	-0.549	-29.3	11.0
Mo	$X_7$	-371	175	3.07	-1.07	-2.64	0.428	16.5	5.56
V	$X_8$	2204	1521	-20.8	19.3	-28.9	24.0	286	-495
Ti	$X_9$	757	-625	-8.44	5.56	-0.0365	-6.19	-44.7	28.3
Al	$X_{10}$	1303	-908	15.2	-9.55	60.6	-36.5	-804	503
Co	$X_{11}$	1874	-412	23.1	-5.63	63.9	-15.2	-1155	270
Nb	$X_{12}$	-291	219	-7.09	5.30	56.3	-6.9	-1529	1610
Cu	$X_{13}$	-84.0	127	4.96	-2.62	-7.59	6.43	-242	124

Studies were performed in the following ranges of thermomechanical factors:  $\xi=0.05-0.3$ ,  $\vartheta=0.1-150 \text{ sec}^{-1}$ ;  $\theta=800-1300 \text{ }^\circ\text{C}$ . Today, however, applied technology, such as thermomechanical rolling, providing deformation of the metal at a temperature less than  $800 \text{ }^\circ\text{C}$ . Also in industrial applications such as the last stand roughing and first group of finishing stands wide-strip hot rolling mill observed reduction of cross-sectional area more than 30%. In this regard, it has been necessary to modify the method of Andreyuk to apply it in a wide range of thermomechanical parameters.

## 1.2. Features of change of thermomechanical factors in the formula of Andreyuk

On an example of steel with chemical composition that produces hot-rolled plates X70 category (**Table 2**) compared the changes of factors  $K_\xi$ ,  $K_\vartheta$  и  $K_\theta$ , calculated according to the formulas of Andreyuk, Pogorzelsky [5] and the Denisov [6].

**Table 2** The ranges of content elements and selected chemical composition of steel X70

	The content of the chemical elements, %											Factors of formula (1)			
	C	Mn	Si	Cr	Ni	Mo	V	Ti	Al	Nb	Cu	$\sigma_0$ , MPa	a	b	c
Min	0.06	1.40	0.08	-	0.03	0.19	-	0.016	0.03	0.03	0.02	78	0.100	0.174	-4.184
Max	0.10	1.92	0.31	0.03	0.21	0.25	0.10	0.017	0.05	0.10	0.03	106	0.132	0.195	-3.144
X70	0.07	1.69	0.22	0.03	0.12	0.22	0.10	0.016	0.04	0.05	0.03	97	0.113	0.189	-3.742

With the selected chemical composition the primary formula of Andreyuk (1), taking into account the relations (2) - (5) takes the following form:

$$\sigma_F = 97 \vartheta^{0.113} (10\xi)^{0.189} (\theta/1000)^{-3.742} \quad (6)$$

Formula of Pogorzelsky:

$$\sigma_F = \begin{cases} 153 \vartheta^{0.2012} \xi^{0.2498} \exp(-0.005\theta) & (\theta > 800^\circ \text{C}) \\ 386 \vartheta^{0.2012} \xi^{0.2498} (0.0305\theta - 0.1953 \cdot 10^{-4} \theta^2 - 10.9) & (\theta < 800^\circ \text{C}) \end{cases} \quad (7)$$

Formula of Denisov:

$$\sigma_F = 2414 \cdot \vartheta^{0.087} \xi^{0.1271} \exp(-0.00286\theta) \quad (8)$$

As is known, thermomechanical factor in some parameter reflects a change of yield stress under the influence of this parameter relatively the base value  $\sigma_{F0}$ . Other thermomechanical parameters should be fixed at levels consistent with the standard test conditions ( $\vartheta_0, \xi_0, \theta_0$ ). Therefore, with the well-known equation

$\sigma_F = \sigma_F(\vartheta; \xi; \theta)$  calculation is performed using the following formulas:

$$K_\vartheta = \frac{\sigma_F(\vartheta; \xi = \xi_0; \theta = \theta_0)}{\sigma_{F0}}; \quad K_\xi = \frac{\sigma_F(\vartheta = \vartheta_0; \xi; \theta = \theta_0)}{\sigma_{F0}} \quad \text{and} \quad K_\theta = \frac{\sigma_F(\vartheta = \vartheta_0; \xi = \xi_0; \theta)}{\sigma_{F0}} \quad (8)$$

For all comparable formulas  $\xi_0 = 0.1$  и  $\theta_0 = 1000$  °C.

However, in the construction of formula (1) the strain was regarded as  $\xi = \varepsilon = (h_0 - h_1)/h_0$ , and in the formulas (7) and (8)  $\xi = \ln(h_0/h_1)$ , where  $h_0$  and  $h_1$  - is the thickness before rolling and after. In all cases, the deformation rate calculated by the formula  $\vartheta = \xi v / \sqrt{R(h_0 - h_1)}$  where  $v$  - is the rolling speed;  $R$  - the roll radius. However, in the formulas (7) and (8)  $\vartheta_0 = 10 \text{ sec}^{-1}$  and Andreyuk indicates  $\vartheta_0 = 1 \text{ sec}^{-1}$ . Despite the fact, that the method of Andreyuk standard test conditions differ substantially lower strain rate values  $\sigma_{F0}$  for the chemical composition of said steel roughly the same (96, 97 and 102 MPa according to formulas Andreyuk, Pogorzelsky and Denisov, respectively).

The graphs that illustrate the changes of thermomechanical factors are shown in **Fig. 1**. Following the method of Andreyuk strain rate factor to 28 - 30 % larger than other methods

(Fig. 1a). The differences between the values  $K_\xi$  (Fig. 1b) in the low strain are minor, but with increasing strain to  $\xi=0.5$  discrepancy reaches 12%. The strain factor of the method of Andreyuk has approximately an average value.

Temperature factor, calculated by formulas Andreyuk and Denisov with variation temperatures vary monotonically (Fig. 1c). Following the method of Pogorzelsky observes an abrupt change  $K_\theta$  at temperature about 800 °C. This fact can be explained by phase transformations in steel. In the temperature range of 1000-1150 °C values  $K_\theta$  for all methods differ insignificantly, and in the range 800-950 °C for the method of Andreyuk characterized by the average value of  $K_\theta$ .

We suppose that for accounting impact of the phase transformation to the Flow Stress formula (1) can be supplemented by a factor  $K_{\gamma\alpha}(\theta) = Q(\theta)/Q(1000)$  where  $Q(\theta)$  and  $Q(1000)$  - values of some physical properties of the steel at a temperature  $\theta$  and 1000°C, respectively. In this case, the formula (1) takes the following form:

$$\sigma_F^* = \frac{\sigma_{F0}}{K_{\gamma\alpha}} \vartheta^a (10 \cdot \xi)^b \left( \frac{\theta}{1000} \right)^c \quad (9)$$

In our study, the factor  $K_{\gamma\alpha}$  is determined based on the known dependence of the specific heat of the steel temperature [7] using piece-wise approximation. For a certain range of temperature  $K_{\gamma\alpha}(\theta) = \beta_0 + \beta_1(\theta/1000)$ ,

where  $\beta_0$  and  $\beta_1$  - factors whose values depend on the type of steel (Table 3)

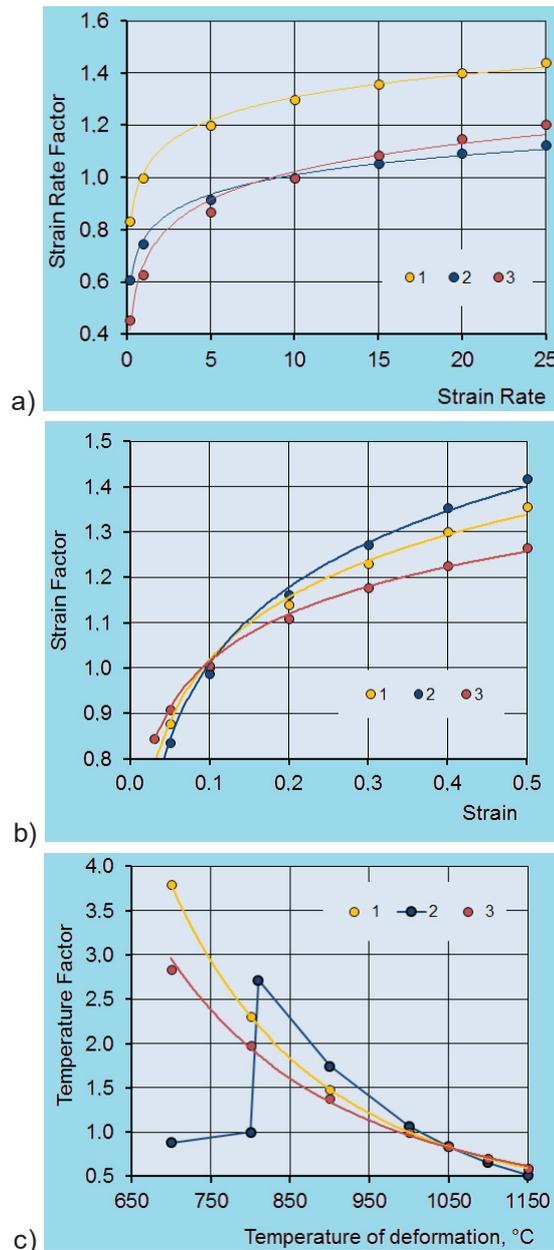


Fig. 1 Variation factors influence strain rate (a), strain (b) and temperature (c): 1, 2, 3 - by the formulas (6), (7) and (8) respectively

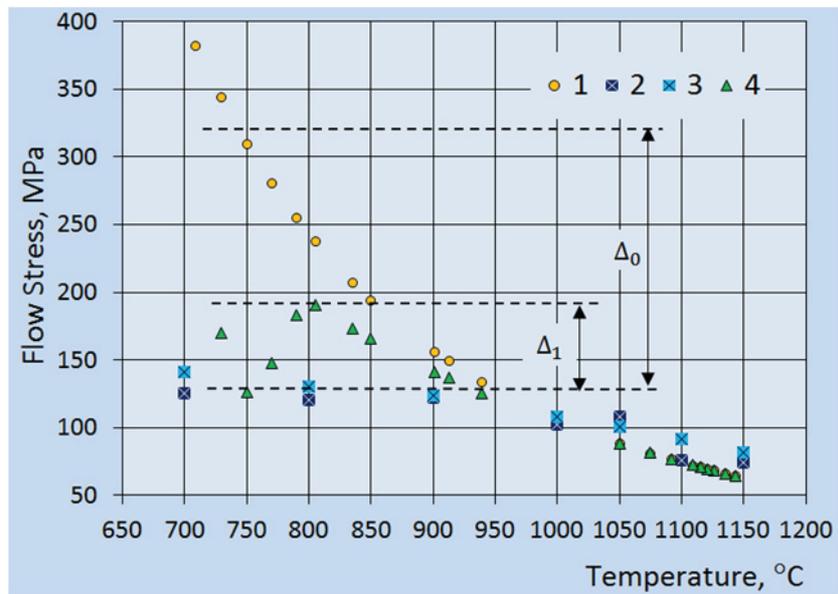
Table 3 The values of factors  $\beta_0$  and  $\beta_1$  for the calculation of the formula (10)

Type of steel	Factors	Temperature, °C						
		Less 700	700 - 750	751-800	801-900	801-950	901 and more	951 and more
Low carbon	$\beta_0$	0.4074	-4.0052	7.0509	-	2.5658	-	0.9845
	$\beta_1$	1.2449	7.5655	-7.176	-	-1.5506	-	0.015
Mild carbon	$\beta_0$	0.4839	-14.886	22.737	2.1511	-	0.7467	-
	$\beta_1$	1.1511	23.117	-27.048	-1.3156	-	0,2467	-
Low alloy	$\beta_0$	0.4619	-12.749	21.385	2.8109	-	0.5819	-
	$\beta_1$	1.3267	20.265	-25.307	-2.0896	-	0.4013	-

**Table 4** A mode rolling steel plate X70

Pass	$\varepsilon$ , %	$v$ , m·s <sup>-1</sup>	$t$ , °C
1	8.0	1.0	1143
2	9.0	1.2	1135
3	9.2	1.3	1126
4	19.5	1.3	1121
5	23.5	2.2	1115
6	26.5	2.5	1109
7	26.7	2.5	1092
8	27.0	2.5	1075
9	13.2	2.5	1050
10	15.4	2.5	940
11	15.5	3.0	913
12	14.3	4.0	902
13	11.4	1.1	850
14	11.4	1.3	835
15	11.1	1.4	805
16	10.6	1.6	790
17	9.1	1.8	770

The results of calculation of Flow Stress in rolling steel X70 for the mode shown in Table 4 (**Fig. 2**).



**Fig. 2** The dependence of Flow Stress on temperature for rolled sheet steel X70:

1, 2, 3 - calculated using the formula (6), (7) and (8); 4 - calculated by the formula (9)

In the calculations using formula (9)  $\sigma_{F0}$ ,  $a$ ,  $b$  and  $c$ , take the same as in formula (6), and the factor  $K_{\gamma\alpha}$  determined for the low alloy steel. The proposed modification of the formula of L.V. Andreyuk improves the

accuracy of calculation of the yield stress at 700-800 °C. The error calculation is reduced by  $\Delta_0=100-250$  to  $\Delta_1=10-25$  MPa.

### 1.3. Identification of models for conditions for hot rolling of heavy plate sheets

To assess the possibility of using a modified formula (9) for practical calculations, considered rolling mill 5000 of heavy plate sheets of micro-alloyed steel grade X70 strength of two different chemical compositions, as well as low-alloy 15CrMo4 and carbon St37.

The average volume for the deformation flow stress found by the unit rolling force  $P_1 = P/b$

$$\bar{\sigma}_F = P_1 / \left[ 1.15 n_\sigma \sqrt{R(h_0 - h_1)} \right], \quad (11)$$

where  $P$  - full rolling force;  $b$  - sheet width;  $n_\sigma$  - stress state factor. Based on the results publication [8]:

$$n_\sigma = \begin{cases} 0.75 + 0.25m & n_{pu} m > 2 \\ 0.5(m + 1/m) & n_{pu} 1 < m \leq 2 \\ 1.25 \ln(1/m) + 1.25m - 0.25 & n_{pu} 0.118 < m \leq 1 \\ 2.57 - 1.44\alpha & n_{pu} m \leq 0.118 \end{cases} \quad (12)$$

where  $m = 2\sqrt{R/(h_0 - h_1)}/(h_0 + h_1)$  - shape factor;  $\alpha = \sqrt{R/(h_0 - h_1)}$  - entering angle.

Total examined 78 passes under the following conditions:  $\theta=750-1050$  °C;  $\xi=0.05-0.25$ ;  $\vartheta=1.5-23$  sec<sup>-1</sup>;  $P_1=9-21$  kN·mm<sup>-1</sup>. A comparison of the Flow Stress  $\sigma_F^*$  and  $\bar{\sigma}_F$  showed the degree of correspondence between them  $R^2=0.826$ , but it revealed a tendency to underestimation of the calculated values (**Fig. 3a**). This fact can be explained by errors formulas which were used to calculate strain and strain rate. To eliminate these errors, supplemented formula (9) by strain and strain rate factor:

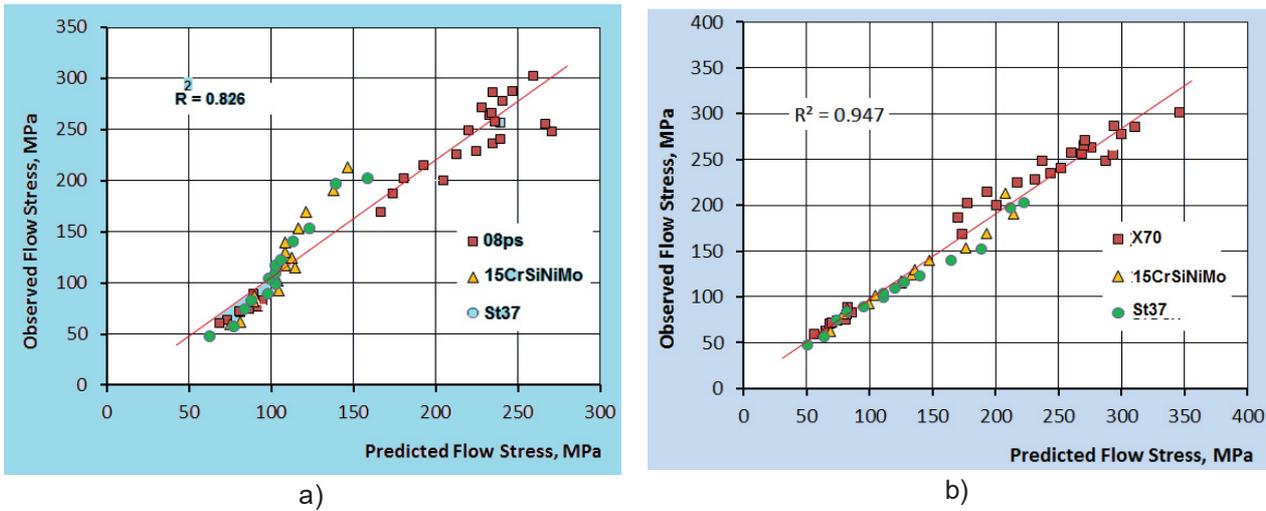
$$K_{\xi\vartheta} = \sigma_F^* / \bar{\sigma}_F. \quad (13)$$

$$\sigma_F^{**} = \frac{\sigma_{F0}}{K_{\gamma\alpha} K_{\xi\theta}} \vartheta^a (10\xi)^b \left( \frac{\theta}{1000} \right)^c \quad (14)$$

To calculate  $K_{\xi\vartheta}$  constructed multiple approximation (confidence level 95%,  $R^2 = 0.894$ )

$$K_{\xi\vartheta} = \xi \left( 0.2012 \frac{R}{h_0} - 12.9257 \right) + \alpha (10.8437 - 22.1367\alpha) - 0.0009\vartheta^2 \quad (15)$$

The degree of compliance  $\sigma_F^{**}$  and  $\bar{\sigma}_F$  increased to  $R^2=0.947$ , tendency to understate the estimated value has disappeared (**Fig. 3b**).



**Fig. 3** Charts of compliance of Predicted and Observed Flow Stress:  
a) calculated using the formula (9); b) calculated using the formula (14)

#### 1.4. Identification of models for conditions for hot wide-strip rolling

Use data on rolling in the finishing mill 415 wide strips of steel grades 08ps, St37, 17MnSi and 15CrSiNiMo. The following conditions are observed:  $\theta=800-1080\text{ }^{\circ}\text{C}$ ;  $\xi=0.055-0.55$ ;  $\vartheta=3-175\text{sec}^{-1}$ ;  $P_1=4.9-29\text{ kN}\cdot\text{mm}^{-1}$ . It is known that the most appropriate for calculating the rolling force in the finishing conditions, is a formula Sims. Therefore, the calculation  $\bar{\sigma}_F$  performed by the formula

$$\bar{\sigma}_F = P_1 / \left[ 1.15 Q_p \sqrt{R(h_0 - h_1)} \right] \quad (16)$$

As a result of the approximation of the graph represented in [9], we obtained the dependence (confidence level 95 %,  $R^2 = 0.991$ ):

$$Q_p = 0.692 + 0.008 \frac{R}{h_1} + 1.984 \varepsilon + 0.016 \varepsilon \frac{R}{h_1} - 2 \cdot 10^{-6} \left( \frac{R}{h_1} \right)^2 - 1.885 \varepsilon^2 \quad (17)$$

Comparison of the results of calculation by the formulas (9) and (16) showed the need to consider the correction factor  $K_{\xi\vartheta}$  separately for low-carbon and low-alloy steel. Regression analysis at a confidence level of 95% received the following approximation (indicators of reliability  $R^2$  of 0.941 and 0.945, respectively):

for low-carbon steel

$$K_{\xi\vartheta} = 1 + 20.1259 \alpha^2 + 3.5773 \xi^2 + 0.0032 \xi R/h_0 - 16.5606 \xi \quad (18)$$

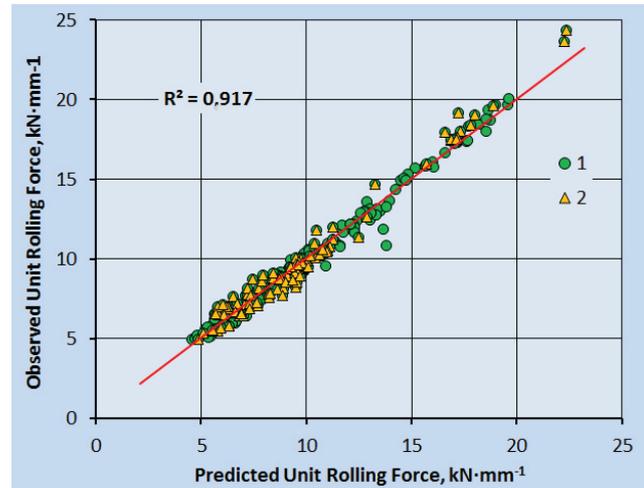
for low-alloy steel

$$K_{\xi\vartheta} = 0.373 - 9 \cdot 10^{-5} \vartheta^2 + 5.19767 \xi - 5.3918 \xi^2 + 0.0034 m R/h_0 \quad (19)$$

Calculation of the Flow Stress of formula (14) with one of the formulas (18) or (19) depending on the type of steel let to achieve the degree of matching observed and predicted rolling force of about 92 % (**Fig. 4**).

## 2. CONCLUSION

Thus, the formula of Andreyuk supplemented by a factor of influence of phase transformations  $K_{\gamma\alpha}$ , as well as correction factor for the strain and strain rate  $K_{\xi\dot{\epsilon}}$ , allows to reduce the lower limit of the confidence interval for the temperature to 700 °C and raise the upper limit of the confidence interval of the strain to 0.5. The dependence of calculation  $K_{\xi\dot{\epsilon}}$  should be constructed taking into account the characteristics of a particular variety of the rolling process. For example, the calculation of the formula (15) allows to reach the degree of compliance with the projected and actual values of the Flow Stress in the rolling heavy plate mill for at least 90 %. The calculation formulas (18-19) allows to reach the degree of compliance with the projected and actual values of the Rolling Force for hot wide-strip rolling of at least 92 %.



**Fig. 4** Charts of compliance of Predicted and Observed Unit Rolling Force:

- 1 - for low-carbon steel;
- 2 - for low-alloy steel

## REFERENCES

- [1] TSELIKOV, A., GRISHKOV, A. *Theory of rolling*. Moscow: Metallurgy, 1970, 358 p.
- [2] PIETRZYK, M., LENARD, J. G. *Thermal-mechanical modeling of the flat rolling process*. Berlin: Springer-Verlag, 1991, 202 p.
- [3] RUMJANCEV, M. The experience of using automatical computer-assistant engineering in developing the working patterns of pickling line tandem cold mill. From *23<sup>rd</sup> International Conference on Metallurgy and Materials METAL 2014*. Brno: TANGER, 2014.
- [4] ANDREYUK, L., TYULENEV, G., PRITZKER, B. Analytical dependence of deformation resistance of steels and alloys on their chemical composition. *Steel*, 1972, No. 6. pp. 522-523.
- [5] POGORZHEL'SKY, V. *Controlled rolling of melting metal*. Moscow : Metallurgy, 1986, 151 p.
- [6] SALGANIK, V., DENISOV S., KRAYNOV V. and others. The resistance deformation of new grades of niobium-containing steels (Steel 05Mn1Nb, 10Mn2VNb, X65 and X70). *Rolling*. 2007, No. 6, pp.15-18.
- [7] *The physical properties of steels and alloys used in the power industry*: Directory / Ed. B.E. Neumark. - M.-L. : Energy, 1967. 240 p.
- [8] POLUHIN, P., KLIMENKO, V., LOPUHIN, V. and others. *Rolling heavy plates*. Moscow: Metallurgy, 1984. 288 p.
- [9] ROBERTS, W. *Hot Rolling of Steel*. New York: Marcel Dekker Inc., 1983, 1024 p.