THE CALCULATION OF STRESS IN THE PROCESSING OF METALS FORMING

Grigory ORLOV, Aleksey POLYAKOV

Ural Federal University, Ekaterinburg, Russian Federation, grorl@mail.ru

Abstract

The urgency of a problem of calculation of the indicators of stress state is considered. The analysis the known methods of calculating the stress in the theory of metal forming was performed. An improved method for calculating the stresses on the basis of finite elements method and the principle of virtual stress is proposed. The process of isothermal axisymmetric deformation is considered. Material adopted isotropic, incompressible, rigidly-visco-plastic. The stress approximated polynomials within the triangular finite element. Unknown coefficients of the polynomials were determined for each element of the solution of systems of the corresponding linear equations. Functional of the Castigliano principle was submitted after the numerical integration of a function of many variables - voltage values at the nodes of elements in a given time. The functional minimization carried out on the computer method of conjugate gradients.

Transition to the next step in time were made by the calculation by the equations of the communication components strain rate tensor on average over the cells with subsequent determination of flow velocity and displacement in the element nodes using formulas of numerical differentiation and current forming.

The algorithm of the calculation in the axially symmetric deformation considered. The method illustrated by compression of cylinder. The comparison of the results with known data shows.

Keywords: Metal forming, stress state, finite elements method, principle of virtual stress

1. INTRODUCTION

The urgency of the problem of calculating the indicators of stress state in processes of treatment of metals by pressure is determined by the necessity of forecasting of processes of destruction and packing metals, healing of defects of origin in the case of processing of cast metal. Widespread variational principles of mechanics are applied mainly for calculating the strain state. The procedure for the calculation of the stress state by strain state includes differentiation of the velocity field that leads to significant errors [1]. The generalized principle of the possible variations of stress and strain states [2] of V. L. Kolmogorov operates with the values of a different order, which causes mathematical difficulties. Also, while minimizing the functional «closure error» on the constraint equations of stress and strain states is minimized invariant, and not in the component sense, therefore, the calculated stress field may not correspond to the reality. When designing the virtual stress fields regardless of virtual strain fields equations of relationship is broken, and can not be met while minimizing that makes it impossible mixing «closure error» to zero. In the present study we examined the statement of the problem and some results of calculation of stress-strain state in axisymmetric plastic deformation, based on the ideas of Castigliano principle and the finite element method.

2. METODS OF RESEARCH

The process of isothermal axisymmetric deformation of body of volume *V* is considered. Made, that the material is isotropic, incompressible, rigidly-visco-plastic with a hardening function in the form of

$$\overline{\sigma} = \overline{\sigma}_0 + f(\varepsilon, \dot{\varepsilon}), \tag{1}$$

where $\overline{\sigma}$ - effective stress; $\varepsilon, \dot{\varepsilon}$ - effective strain and strain rate; $\overline{\sigma}_0$ - yield stress.

We will pass also the hypothesis of a «unified curve», so that the dependence (1) can be obtained from experiments on simple loading. Suppose, that the mass and inertia forces are absent.

The equilibrium equations have the form of taking into account the adopted assumptions:

$$\begin{cases} \partial\sigma_{rr}/\partial r + \partial\sigma_{rz}/\partial z + (\sigma_{rr} - \sigma_{\varphi\varphi})/r = 0; \\ \partial\sigma_{rz}/\partial r + \partial\sigma_{zz}/\partial z + \sigma_{rz}/r = 0; \end{cases} \qquad (2)$$

To determine the actual stresses we use the Castigliano principle, and for the formation of a statically possible stress fields - of the ideas of the finite elements method. After integrating the functionality of the Castigliano principle can be reduced to the functions of many variables - voltage values at the nodes of the elements.

For the problem solving in time carried out step-by-step procedure of calculation. The time step is chosen from the condition:

$$\Delta t^{s+1} = \max_{i=1}^{k}(\Delta\varepsilon_i^s)/\max_{i=1}^{k}(\dot{\varepsilon}_i^s), \qquad (3)$$

where $\Delta\varepsilon_i^s$ - increase of effective strain in *i*-element at *s*- time step; k - quantity of elements.

The following sequence of operations are carried out to go to the next step in time:

strain-rate tensor components average elements were calculated by the constitutive equations $\dot{\varepsilon}_{ij} = (3\dot{\varepsilon}/2\overline{\sigma})s_{ij}$ (where $\dot{\varepsilon}_{ij}$ - strain rate tensor; $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ - deviatoric stress);

the velocity of flow in the nodes of elements were determined using formulas of numerical differentiation; for example:

$$\dot{\varepsilon}_{zz} \approx \Delta\upsilon_z/\Delta z = \left[-\sum_{i=1}^{3}(\upsilon_{z_{i+1}} + \upsilon_{z_i})(r_{i+1} - r_i)\right] \Big/ \left[\sum_{i=1}^{3}(r_{i+1} - r_i)(z_{i+1} - z_i)\right]. \qquad (4)$$

This expression can be reduced to an equation with one unknown quantity for every element, using the boundary conditions and symmetry axes.

New coordinates of nodes after the *i*-th step following: $z_i = z_{i-1} + \Delta t\upsilon_z$; $r_i = r_{i-1} + \Delta t\upsilon_r$, where υ_z, υ_r - velocities of flow.

The proposed algorithm is considered on the example of upset cylinder. We consider a quarter of the cylinder, which in accordance with the method of finite elements can be divided into *K*×2*N* triangular elements of the ring.

The boundary conditions were considered as following. On the free surface S_1 we have $\sigma_{ij}n_j = 0$, where n_j - direction cosines. On the surface S_2 (**Fig. 1**) a full adhesion is implemented: $\upsilon_z = -\upsilon^*; \upsilon_r = 0$.

We have also because of axisymmetric deformation: $\sigma_{rr} = \sigma_{\varphi\varphi}; \sigma_{rz} = 0$ on the axis *z*; $\sigma_{rz} = 0$ on the axis *r*.

Given these assumptions functional Castigliano principle will have the form:

$$J = \int_V \left[\int_{\overline{\sigma}_0}^{\overline{\sigma}} \dot{\varepsilon}(\sigma)d\sigma\right]dV - \int_{S_2} \upsilon^*\sigma_{zz}\big|_{z=h}ds, \qquad (5)$$

where $\dot{\varepsilon}(\sigma)$ - inverse function from (1).

According to the Castigliano principle the actual field of stresses must satisfy the boundary conditions on the S_1, equilibrium equations (2) and report the minimum of the functional (5).

Within the element the stresses were approximated by polynomials of the form:

$$\sigma_{rz} = rz(b_1 + b_2 r + b_3 z + q_3 rz); \tag{6}$$

$$\sigma_{rr} = a_1 + a_2 r + a_3 z \,. \tag{7}$$

The expressions for σ_{zz} and $\sigma_{\varphi\varphi}$ were received from the solution of equilibrium equations (2) after substitution polynomials (6), (7):

$$\sigma_{zz} = q_1 + q_2 r - q_3 r z^3 - z^2 (b_1 + 3b_2 r/2 + 2b_3 z/3); \tag{8}$$

$$\sigma_{\varphi\varphi} = a_1 + 2a_2 r + a_3 z + r \times (b_1 r + b_2 r^2 + 2b_3 rz - 2q_3 z r^2). \tag{9}$$

Thus, the selected stress field satisfies of the equilibrium equations (2) and conditions $\sigma_{rr} = \sigma_{\varphi\varphi}$, $\sigma_{rz} = 0$. The boundary condition $\sigma_{ij} n_j = 0$ has been satisfied directly. Unknown coefficients of approximating polynomials (6)...(9) were determined for each element of the solution of linear systems of three equations. For example, the coefficients b_1, b_2, b_3 were determined from the solution of the system $\sigma_{rzK} = r_K z_K (b_1 + b_2 r_K + b_3 z_K + q_3 r_K z_K)$ for one of the elements, where σ_{rzK} - levels of stresses in nodes; r_K, z_K - coordinates of nodes; K=1...3 - numbers of nodes.

The coefficients q_1, q_2, q_3 were determined after substitution b_1, b_2, b_3 in expression (8) and solving the system $\sigma_{zzK} = q_1 + q_2 r_K - q_3 r_K z_K^3 - z_K^2 (b_1 + 3b_2 r_K / 2 + 2b_3 z_K / 3)$. The coefficients a_i were determined similarly.

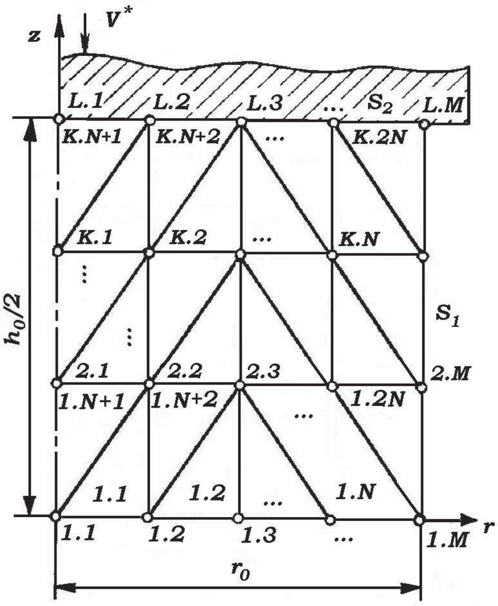

Fig. 1 Scheme of decomposing of a quarter of the cylinder into triangular elements

Functional (5) was presented in the form of function of several variables after the substitution of expressions (6)...(9) and integration. Thus, the problem of minimizing the functional has been reduced to the minimization of a function of many variables - values of the stresses at the nodes of the elements.

Transition to the subsequent step in time was made according to the above-mentioned scheme. The calculation of υ_z was carried out consistently, starting with the elements adjacent to the axis r, where υ_z =0. For example, the velocity of the node 2.2 was determined by two well-known velocities in the nodes 1.1 and 1.2 of element 1.1 (**Fig. 1**). Similarly velocity υ_r was counted starting from the elements, adjacent to the axis z, where υ_r =0. On the surface the velocities were accepted: $\upsilon_z = -\upsilon^*$, υ_r =0 in accordance with the boundary conditions. The ambiguous definition of velocities are possible in some nodes in this procedure of calculation. The «closure error» was 7...10 %, which is within the error of step by step calculation.

After defining new coordinates of nodes and curving of side face (free surface), the boundary condition $\sigma_{ij} n_j = 0$ were used in the form

$$\sigma_{rz} = -\sigma_{zz} n_z / n_r ;\ \sigma_{rr} = \sigma_{zz} n_z^2 / n_r^2 , \tag{10}$$

where direction cosines n_z, n_r in the nodes of a free surface are defined as the average direction cosines adjacent faces elements. Thus, on the free surface stresses σ_{rz} and σ_{rr} were not varied.

Account of conditions on axes and conditions (10) has allowed to allocate the dependent variables and reduce the number of unknowns, defined by the formula *X=3ML-[M+L+(L-1)(M-2)]*, where *M* and *L* is the number of nodes in the *r* and *z* axes.

3. RESULTS AND DISCUSSION

Implementation of the above algorithm is considered for the example of the process of axial-symmetric upset of cylinder (**Fig. 1**) size of h_0=1.8 m; r_0=0.6 m; speed tool υ^* = 0.3 m/s; the temperature field in the uniformly *T*=1200°*C*; material - steel *C0.9Cr2MoV*.

In addition to components of the stress tensor, were calculated the hydrostatic stress $\sigma = \sigma_{ii} / 3$ and an indicator of the stress state $k = \sqrt{3}\sigma / \overline{\sigma}$, where $\overline{\sigma}$ - effective stress. The resulting decisions distribution through the volume σ_{zz} and *k* for the relative compression 19.8 % is shown in **Fig. 2**, where *R* and *h*, respectively current radius and a half the height of the cylinder.

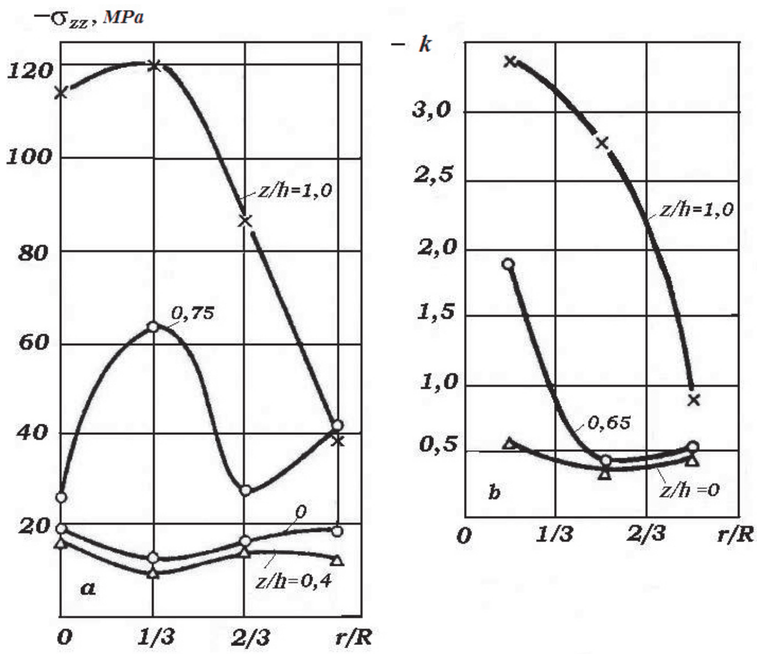

Fig. 2 Distribution through the volume of the cylinder of σ_{zz} (a) and *k (b)*

Comparison with known experimental data showed that the calculated diagram of σ_{zz} on a contact surface has the same character as that of experimental data of upset. The estimated values of the *k* on the free surface differs from the experimental [3,4] not more than 10 %.

The computer program is developed for problem solving with minimization by the method of conjugate gradients.

CONCLUSION

So, the alternative method for calculating the stresses was discussed in this article. The method is based on ideas of finite elements method and the Castigliano principle of virtual stress.

The process of isothermal axisymmetric deformation is considered. Material adopted isotropic, incompressible, rigidly-visco-plastic. The stresses were approximated by polynomials within the triangular finite element. Unknown coefficients of the polynomials were determined for each element of the solution of systems of the corresponding linear equations. Functional of the Castigliano principle was submitted after the numerical integration of a function of many variables - voltage values at the nodes of elements in a given time. The functional minimization carried out on the computer method of conjugate gradients.

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