HOW TO SELECT THE PORTFOLIO OF SUPPLIERS USING THE OUTRANKING MULTI-ATTRIBUTE DECISION MAKING METHOD

František ZAPLETAL

VSB - Technical university of Ostrava, Faculty of Economics, Ostrava, Czech Republic, EU,
frantisek.zapletal@vsb.cz

Abstract

Suppliers are an inevitable part of a logistics chain. It is crucial for each company to select the best possible suppliers. Suppliers' selection is a typical problem of multi-attribute decision making (MADM). All traditional methods for multi-criteria decision making have already been used to evaluate suppliers. However, very often the ranking of (potential) suppliers is not the final desirable result, because not only the best one, but a set of suppliers must be chosen. An intuitive way would be to select the first *n* suppliers in the ranking. But, this study shows, that this approach can lead to distorted results. In this paper, the PROMETHEE outranking method of MADM combined with mixed integer programming is used to find the optimal portfolio of suppliers under various constraints (number of suppliers, demand for different types of products, available stocks of suppliers, limited budget of the company, etc.). The proposed application is demonstrated using the numerical example.

Keywords: Suppliers, portfolio, PROMETHEE, outranking methods

1. INTRODUCTION

Quality of suppliers plays a crucial role in competitiveness and success of each production company. The suppliers' selection is a strategic decision problem, which deserves a great attention. And since every strategic decision should be supported by quantitative analysis, the same applies for the suppliers' selection too. But, the use of quantitative methods of decision making to find the best supplier from the set of potential suppliers is not novel at all. All the currently most popular MADM (multi-attribute decision making) methods have already been used to handle this problem - analytic hierarchy process (AHP) [1,2], analytic network process (ANP) [3,4], TOPSIS [5], ELECTRE [6,7], or PROMETHEE [8,9], or their combinations [10,11]. If the aim is to find the best one supplier, or to get the ranking of the suppliers, the mentioned approaches are highly suitable.

However, sometimes, a company must deal with the situation where more than one supplier must be selected for some reason like the risk diversification, spatial or time disproportion, stock quantity, etc. In this case, the company must choose the best portfolio of suppliers under some criteria and constraints. This kind of problem is by far not as popular as the one above. The authors of [12] have presented the model using the combination of the ANP and Data Envelpment Analysis (DEA), and [13] have come with the solution using genetic algorithms and optimisation methods.

The aim of this paper is to provide the model for suppliers' portfolio selection based on the PROMETHEE method and mixed integer programming (MIP). The PROMETHEE method is a MADM method, which is user-friendly with very transparent computational procedure. Namely, I use its PROMETHEE V extension presented by [14].

Some intuitive heuristics can suggest to use any of the methods for suppliers' ranking and then to put the first into the portfolio. If some required constraint is not met, then the second supplier according to the ranking is added, and so on, until all the constraints are satisfied. However, such approach can result in a non-optimal decision. The constraints can cause that a combination of the alternatives with lower rankings can be better, than some higher-ranked alternative from the perspective of feasibility.

I demonstrate the PROMETHEE-based approach using a numerical example where the optimal portfolio is found under the following constraints: the suppliers have the limited available quantities of the products, the suppliers provide their goods at different prices and the budget of the decision maker is limited, all the demand of the deciding company must be satisfied and the company decides also on how many suppliers should be in the optimal portfolio.

The results show that the decision problem is not trivial and that the used approach is suitable to solve the suppliers' portfolio selection problem.

The rest of the paper is organised as follows. Section 2 recalls the methodology of the PROMETHEE method and its extension called PROMETHEE V for the portfolio selection. In Section 3, the numerical example is described, solved and discussed. Conclusions of this research are provided in Section 4.

2. THE PROMETHEE V METHODOLOGY

The PROMETHEE method has been developed [15] in efforts to present the easily understandable and user-friendly method for ranking alternatives. And the fact that they were successful is proved by [16] and their review of published applications of the PROMETHEE method. The algorithm of the PROMETHEE method for the complete ranking of alternatives can be summarised into the following steps:

- Preference degrees $P_i(A_x, A_y) \in [0,1]$ are calculated for all pairs of alternatives A with respect to each criterion $i = 1, \ldots, k$ using preference functions P_i (this function assigns a preference degree to each possible difference in performance values). The preference degree says, how much the decision maker prefers an alternative with better performance in the given criterion to the one with worse performance.
- The preference degrees are aggregated to preference indices expressing how much the decision maker prefers one alternative to another. This is done using the sum product of preference degrees and weights.
- The preference indices are aggregated to positive and negative flows ($\phi^+ \in [0,1], \phi^- \in [0,1]$) of each alternative, see Eq. (1) and (2). The positive flow of an alternative is a mean value of the preference indices comparing this alternative with the others (how much better is the alternative than the others). The other way around, the negative flow of an alternative is a mean value of the preference indices the remaining alternatives with the one under evaluation (how much worse is the alternative than the others).
- Due to the fact that the ranking using only the positive and negative flows provides only a partial ranking, these partial flows must be aggregated to the net flows $\phi \in [-1,1]$, see Eq. (3).

The calculations of the described algorithm can be shortly written as follows:

$$\phi^+(A_t) = \frac{\sum_{j=1, j\neq t}^{s} w_i \cdot P_i(v_{i,t} - v_{j,t})}{s-1}, \tag{1}$$

$$\phi^-(A_t) = \frac{\sum_{j=1, j\neq t}^{s} w_i \cdot P_i(v_{j,t} - v_{i,t})}{s-1}, \tag{2}$$

$$\phi(A_t) = \phi^+(A_t) - \phi^-(A_t), \tag{3}$$

where w_i is the weight of the i-th criterion ($\sum_{i=1}^{k} w_i = 1$), $P_i(v_{j,t} - v_{i,t})$ stands for the preference degree at which A_j is preferred to A_i considering the i-th criterion, s is a number of alternatives, k represents the number of criteria, $v_{i,t}$ stands for the performance value of the i-th alternative with respect to the t-th criterion. The authors of [15] have defined the properties of the preference function. A decision-maker can choose any non-decreasing function P (the greater difference in performances, the greater (or equal) preference strength in favour of the better alternative) with $P(x) = 0$ for $x \leq 0$, the domain of all real numbers ($x \in \mathbb{R}$) and the range $P(x) \in [0,1]$. In order to make the choice of preference functions simpler for decision-makers, [15] have proposed some predefined functions' shapes. But, by far the most common shape is the linear one, which allows to consider too small differences in performance values as negligible using the indifference threshold p and too big differences are omitted using the preference threshold q, see **Figure 1.**

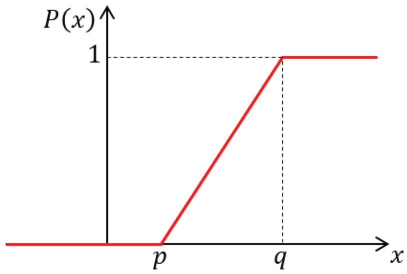

Figure 1 Linear and usual shapes of the preference functions [own processing]

The selection of the optimal portfolio is done using the mixed integer programming (MIP) model with the flows from the PROMETHEE in its objective function [14]:

$$\max_{x \in \{0,1\}^{n\times 1}} \boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{x} \qquad \text{s.t.} \quad \mathbf{A}\boldsymbol{x} \leq \boldsymbol{b}, \tag{4}$$

where the vector of binary variables $\boldsymbol{x}$ determines, if the alternative is included in the portfolio ($x = 1$), or it is not included ($x = 0$), $\boldsymbol{\phi} \in [-1,1]^{s\times 1}, \boldsymbol{\phi}^{\mathrm{T}} = (\phi(A_1), \phi(A_2), \dots, \phi(A_s))$ is the vector of the net flows from the PROMETHEE ranking algorithm, $\mathbf{A} \in \mathbb{R}^{n\times s}$ and $\boldsymbol{b} \in \mathbb{R}^{n\times 1}$.

The model (4) can be easily solved by any software for mathematical programming like GAMS or AMPL. Apart from the optimal portfolio, which is the optimal solution of (4), a *c*-optimal portfolio can also be searched. The *c*-optimal portfolio is the optimal solution of (4) with one additional constraint:

$$\boldsymbol{e}^{\mathrm{T}}\boldsymbol{x} = c, \tag{5}$$

where $\boldsymbol{e} \in \mathbb{R}^{s\times 1}$ is the vector of ones and c is a scalar determining the size of the portfolio. The *c*-optimal portfolios are beneficial for the analysis how the optimal portfolio of alternatives differs with its size.

3. NUMERICAL EXAMPLE

The methodology described in Section 2 is applied on the numerical example of the suppliers' ranking provided by [17]. The problem includes 18 suppliers (S1-S18) and 5 criteria for evaluation (C1: supply variety (pieces), C2: quality (% of non-defect products), C3: distance (km), C4: delivery (% of products delivered in time), C5: price index (%). Due to the limited page capacity of this paper, the performances of the suppliers in the involved criteria cannot be shown directly in this paper, but it can be found in [17]. To perform the PROMETHEE ranking analysis, additional input data must be considered. Namely, the weights of the criterion w_i are taken the same for all 5 criteria, i.e. all equal 0.2. All the criteria are carried out by the linear preference function type (see **Figure 1**) and the used thresholds are shown in **Table 1**. The PROMETHEE ranking has been calculated using Eqs. (1-3) and the results can be found in **Table 2** (the alternatives are ordered with respect to their net flows here).

Table 1 Indifference $\boldsymbol{p}$ and preference $\boldsymbol{q}$ thresholds of the used preference functions [own study]

	C1	C2	C3	C4	C5
p	5	0	30	0	0
q	20	5	1000	20	11

Table 2 Results of the PROMETHEE ranking [own study]

Rank	Supplier	Phi	Phi+	Phi-	Rank	Supplier	Phi	Phi+	Phi-
1	S15	0.4008	0.4373	0.0365	10	S1	-0.0167	0.129	0.1457
2	S17	0.3953	0.4363	0.041	11	S9	-0.0438	0.0869	0.1307
3	S10	0.2204	0.03429	0.1225	12	S16	-0.0671	0.0964	0.1635
4	S5	0.1546	0.2274	0.0728	13	S3	-0.0994	0.0686	0.168
5	S8	0.0915	0.2045	0.1131	14	S2	-0.1399	0.0822	0.2221
6	S11	0.0794	0.1725	0.0931	15	S18	-0.1866	0.0494	0.236
7	S6	0.0513	0.2333	0.1819	16	S4	-0.2879	0.0405	0.3285
8	S13	0.0498	0.1444	0.0946	17	S7	-0.3005	0.0363	0.3368
9	S12	0.0246	0.1392	0.1146	18	S14	-0.3258	0.0454	0.3712

Table 3 Available quantity of the products by the suppliers and the unit prices (in upper indices) [own study]

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
P1	10^{6}	0	0	20^{8}	0	0	0	0	0	0	0	0	30^{7}	0	0	0	0	0
P2	0	0	30^{5}	40^{6}	0	0	0	30^{7}	0	0	80^{4}	0	0	60^{6}	0	0	0	0
P3	0	0	0	50^{7}	0	0	20^{6}	0	0	0	0	0	50^{9}	0	0	0	0	0
P4	0	20^{2}	0	0	100^{3}	0	0	0	0	0	50^{1}	0	0	80^{2}	0	0	100^{3}	0
P5	0	0	0	0	40^{5}	30^{5}	0	0	40^{4}	0	0	0	0	0	50^{5}	0	30^{4}	0
P6	0	25^{6}	0	0	50^{5}	0	0	60^{5}	0	60^{4}	0	0	0	0	0	60^{3}	0	0
P7	0	0	0	0	0	0	0	50^{9}	0	60^{10}	0	50^{8}	0	0	50^{9}	0	0	0
P8	0	0	0	0	0	0	90^{4}	0	0	80^{5}	0	0	0	0	0	0	90^{5}	0
P9	0	0	0	0	0	0	0	0	0	120^{4}	0	0	0	0	80^{5}	0	0	70^{3}
P10	0	0	0	0	0	0	0	0	0	0	80^{4}	0	0	0	0	0	0	100^{3}

Now, let us assume that the company uses 10 different inputs for production. But none of the suppliers under evaluation is able to supply all the inputs on its own. Thus, the company wants to find such portfolio of suppliers, which will supply the required inputs all together and with a maximum sum of the net flows. In **Table 3**, the amounts of 10 considered inputs (P1 to P10) and the available amounts by all the suppliers are displayed. Unit prices for the suppliers and their products can also be found in **Table 3** (they are written as the upper indices). The company needs the following quantities of the inputs: (15; 60; 70; 120; 80; 70; 40; 100; 100; 100) units.

To find the optimal suppliers' portfolio, the following model based on (4), is solved:

$$\max_{x\in\{0,1\}^{n\times 1}} \sum_{j=1}^{18} \phi_j x_j, \qquad i = 1,\dots,10, \tag{6a}$$

$$\text{s.t.} \quad \sum_{j=1}^{18} y_{ij} = d_i, \qquad i = 1,\dots,10, \tag{6b}$$

$$y_{ij} \le s_{ij}, \qquad i = 1,\dots,10, j = 1,\dots,18, \tag{6c}$$

$$\sum_{i=1}^{10} \sum_{j=1}^{18} p_{ij} y_{ij} \le b, \tag{6d}$$

$$\sum_{i=1}^{10} y_{ij} \le M(1 - a_j), \qquad j = 1,\dots,18, \tag{6e}$$

$$a_j + x_j = 1, \qquad j = 1,\dots,18, \tag{6f}$$

$$\sum_{j=1}^{18} x_j = c, \tag{6g}$$

$$x_j, a_j \in \{0,1\} \qquad j = 1, \dots, 18, \qquad (6h)$$

$$y_{ij} \geq 0 \qquad i = 1, \dots, 10, j = 1, \dots, 18, \qquad (6i)$$

where d_i is the demand for the i-th product, s_{ij} stands for the stock level of the i-th product by the j-th supplier, p_{ij} denotes the unit price for the i-th product provided by the j-th supplier, b is the budget of the supplied company and y_{ij} is the variable representing the amount of the i-th product delivered by the j-supplier. The constraints (6b) guarantee that the demand of the company is fully satisfied, (6c) do not allow to exceed the stock capacity of the suppliers, (6d) is a budget constraint and (6g) determines the exact number c of the suppliers in the portfolio. The constraints (6e) and (6f) are the result of linearisation of the following implication: "If the company buy any unit of any product from the j-th supplier, the binary variable x_j equals 1. Otherwise, x_j equals 0". These constraints use M as a sufficiently great prohibitive constant and a_j is a dummy binary variable. The model has 216 variables (180 real variables and 36 binary variables) and 228 constraints (excluding non-negativity constraints and binary constraints) in total.

The model (6) is solved using the GAMS software and its MIP solver. The model is solved without the (6g) constraint to find the upper bound c^u for c, and then for $c = \{c^l, c^l + 1, \dots, c^u\}$, where c^l is the least c, for which (6) is still feasible.

The model (6) has been solved by the computer with I7 Intel processor 2.59GHz, 8GB RAM and Windows 10 x64 OS. Each run of the model has lasted from 38 to 40 seconds.

The optimal solution for (6) without the constraint (6g) involves 11 suppliers in the optimal portfolio, see **Table 4**. As expected, all the suppliers with a positive value of the net flow are included in this portfolio (they improve the value of the objective function). Therefore, the upper bound for the number of suppliers in the portfolio (c^u) is equal to 11. The results of (6) for $c = 7,8,9,10,11$ are shown in **Table 4** (for $c < 7$, the model has no feasible solution). One can expect that when c is decreased by 1, one (the least suitable) supplier will be excluded from the portfolio and the remaining suppliers will still be selected. But, the results in **Table 4** shows that this is not true in general, see the supplier S11 that is in the *c*-optimal portfolios for $c = \{7,9,10,11\}$, but not for $c = 8$.

Due to the fact that no better value of (6a) than the one for $c = c^u$ can be reached, it is clear that the optimal values (6a) must decrease with c. It is worth noting that the total costs related with the *c*-optimal portfolios can either decrease with decreasing c ($c = 9 \rightarrow 8$), or increase ($c = 11 \rightarrow 10$, or $8 \rightarrow 7$), or even maintain at the same level ($c = 10 \rightarrow 9$). The reason is that the costs are not involved in the objective function of the model.

Table 4 *c*-optimal portfolios for ***c*** from 7 to 11, their objective function's value (OF) and the total costs

c	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	OF	Costs
7	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	0	0	1	0.42	3265
8	0	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	1	1	0.84	3525
9	0	0	0	1	1	0	0	1	0	1	1	0	1	0	1	0	1	1	0.92	3285
10	0	0	0	1	1	1	0	1	0	1	1	0	1	0	1	0	1	1	0.97	3285
11	0	0	0	1	1	1	0	1	0	1	1	1	1	0	1	0	1	1	0.99	3245

4. CONCLUSION

This paper demonstrates the way, how the PROMETHEE ranking method combined with mixed integer programming can be used to find the optimal portfolio of suppliers. The suitability of adding a supplier to the portfolio is assessed using the PROMETHEE ranking, which is calculated for any selected criteria of evaluation. Using the numerical example, I have shown that this transparent and easy-to-calculate method provides non-trivial results and that it is suitable for solving the given economic problem. The further research should be focused on application of the presented method to some real-life example.

ACKNOWLEDGEMENTS

This work was supported by the ESF in "Science without borders" project, reg. nr. CZ.02.2.69/0.0/0.0/16_027/0008463 within the Operational Programme Research, Development and Education.

REFERENCES

[1] KAHRAMAN, Cengiz, CEBECI, Ufuk and ULUKAN, Ziya. Multi-criteria supplier selection using fuzzy AHP. *Logistics information management*. 2003. vol. 16, no. 6, pp. 382-394.

[2] CHAN, Felix TS, KUMAR, Niraj, TIWARI, Manoj Kumar, LAU, Henry and CHOY, K L. Global supplier selection: a fuzzy-AHP approach. *International Journal of production research*. 2008. vol. 46, no. 14, pp. 3825-3857.

[3] GENCER, Cevriye and GÜRPINAR, Didem. Analytic network process in supplier selection: A case study in an electronic firm. *Applied mathematical modelling*. 2007. vol. 31, no.11, pp. 2475-2486.

[4] DARGI, Ahmad, ANJOMSHOAE, Ali, GALANKASHI, Masoud Rahiminezhad, MEMARI, Ashkan and TAP, Masine Binti. Supplier selection: A fuzzy-ANP approach. *Procedia Computer Science*. 2014. vol. 31, pp. 691-700.

[5] BORAN, Fatih Emre, GENÇ, Serkan, KURT, Mustafa and AKAY, Diyar. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Systems with Applications*. 2009. vol. 36, no. 8, pp. 11363-11368.

[6] DE BOER, Luitzen, VAN DER WEGEN, Leo and TELGEN, Jan. Outranking methods in support of supplier selection. *European Journal of Purchasing & Supply Management*. 1998. vol. 4, no. 2-3, pp. 109-118.

[7] SEVKLI, Mehmet. An application of the fuzzy ELECTRE method for supplier selection. *International Journal of Production Research*. 2010. vol. 48, no. 12, pp. 3393-3405.

[8] SENVAR, Ozlem, TUZKAYA, Gülfem and KAHRAMAN, Cengiz. Multi criteria supplier selection using fuzzy PROMETHEE method. *In: Supply chain management under fuzziness*. Springer, Heidelberg. 2014. p. 21-34.

[9] CHEN, Ying-Hsiu, WANG, Tien-Chin and WU, Chao-Yen. Strategic decisions using the fuzzy PROMETHEE for IS outsourcing. *Expert Systems with Applications*. 2011. vol. 38, no. 10, pp. 13216-13222.

[10] DAĞDEVIREN, Metin. Decision making in equipment selection: an integrated approach with AHP and PROMETHEE. *Journal of intelligent manufacturing*. 2008. vol. 19, no. 4, pp. 397-406.

[11] LIAO, Chin-Nung and KAO, Hsing-Pei. An integrated fuzzy TOPSIS and MCGP approach to supplier selection in supply chain management. *Expert Systems with Applications*. 2011. vol. 38, no. 9, pp. 10803-10811.

[12] [12] ABDOLLAHI, Mohammad, ARVAN, Meysam and RAZMI, Jafar. An integrated approach for supplier portfolio selection: Lean or agile? *Expert Systems with Applications*. 2015. vol. 42, no. 1, pp. 679-690.

[13] DING, Hongwei, BENYOUCEF, Lyès and XIE, Xiaolan. A simulation optimization methodology for supplier selection problem. *International Journal of Computer Integrated Manufacturing*. 2005. vol. 18, no. 2-3, pp. 210-224.

[14] BRANS, J. Pierre and MARESCHAL, Bertrand. PROMETHEE V: MCDM problems with segmentation constraints. INFOR: *Information Systems and Operational Research*. 1992. vol. 30, no. 2, pp. 85-96.

[15] MARESCHAL, Bertrand, BRANS, Jean-Pierre and VINCKE, Philippe. *PROMETHEE: A new family of outranking methods in multicriteria analysis*. ULB-Universite Libre de Bruxelles, 1984.

[16] BEHZADIAN, Majid, KAZEMZADEH, Reza Baradaran, ALBADVI, Amir and AGHDASI, Mohammad. PROMETHEE: A comprehensive literature review on methodologies and applications. *European journal of Operational research*. 2010. vol. 200, no. 1, pp. 198-215.

[17] NG, Wan Lung. An efficient and simple model for multiple criteria supplier selection problem. *European journal of operational research*. 2008. vol. 186, no. 3, pp. 1059-1067.