THE COMPLEXITY SHAPING ALGORITHM FOR THE COOPERATION OF CONVERGENT PRODUCTION SYSTEMS

Bożena ZWOLIŃSKA

AGH University, Faculty of Mechanical Engineering and Robotics, Cracow, Poland, EU, bzwol@agh.edu.pl

Abstract

The complexity of manufacturing systems is a result of the emergence, which unconditionally relies on the qualities of the analyzed manufacturing system, the type and scale of production and the features and kind of manufactured final products. For realistic production systems formally defining their complexity is difficult and ambivalent. This article discusses the topic of formalizing various types of manufacturing systems complexity and presents an algorithm for evaluating the system basing on cooperation complexity. The presented algorithm applies to systems of convergent production characteristics, for which the evaluation of existing codependences is an NP - hard problem. The formalized algorithm is developed basing on the number of existing relations (both asymmetrical and symmetrical) defined on chosen subsystems of a complex manufacturing structure. The decomposition stage has been conducted as per the principles of the general systems theory. The final part of the article presents an analysis of the correctness of the formalized algorithm in an extreme example, i.e.: two systems S_I and S_{II} have been compared, the structure of which comprised respectively of n (for S_I) and $n+1$ (for S_{II}) defined subsystems which have a certain number of relations of one type established.

Keywords: Convergent production systems, cooperation complexity

1. INTRODUCTION

Progressing globalization and systematically expanding the markets are currently no longer the only determinant assuring a company`s success. Next to development and territorial expansion in order to maintain competitiveness, it is necessary to introduce innovative solutions. The never ending technological progress stimulates the creation of new or improved products with a constantly shorter life cycle. A brand which until recently was the synonym for quality and prestige now gives way to more interesting inventions and technological nuances. In an environment of unforeseeable competition staying afloat on the market for multiple years is an extremely challenging task. Each enterprise aware of the risk of bankruptcy, in addition to monitoring the finances of its own investments, must constantly benchmark the development of its rivals, observe joint venture initiatives and monitor new start-up`s. These however are not all the risks increasing the chances for failure. The until recently popular idea of "*lean manufacturing*" originating from *TPS - Toyota Production System* is slowly losing its significance and is being pushed aside by the new industrial revolution. *Industry 4.0* has become the philosophy of today. The fourth industrial revolution blends the real world with the virtual one. Reaching level 4.0 means that a company has achieved integral coherence and synergy of all of its areas. *Industry 4.0* is not only digitization and processing huge amounts of information and metadata in cyberspace. Industry 4.0 is also more than *Digital Product Engineering*, *Digital Manufacturing Engineering*, or the *Digital Factory. Industry 4.0* is unifying all the elements of a company, both real and virtual, into one integral body.

According to the general theory of complex systems, a system is a collection of elements and relations existing between these elements. Therefore each production company is usually a complex system and the complexity level depends on the evaluation criteria. The complexity of systems (both material and immaterial) is a result of emergence, which unconditionally relies on the characteristics of the system`s components. Formalizing the definition complexity of a manufacturing system includes many aspects, i.a.: structural complexity, functional complexity, cooperation complexity, flow complexity and total complexity. Determining the complexity level of a manufacturing system is a difficult task, ambiguous and depends on the assumed evaluation algorithm. For the purposes of this article, the complexity of the manufacturing system and its assigned subsystems has been determined by using the following parameters:

- the number of relations between the elements of the discussed system,
- the number of assigned subsystems,
- the level of relation complexity between the assigned elements (subsystems).

The complexity of a system is subjective, however it is unconditionally dependent objective evaluations. While determining the manufacturing system`s complexity, depending on the assumed evaluation criteria, it is necessary to take into account the following: the complexity of the system`s structure, the complexity of functions performed by the system, the complexity of relations between the elements which make up the system and/or it`s environment, the number of elements and the number of comprising relations of the discussed system. The system`s complexity level is also influenced by the complexity of the final product manufactured within the discussed *PS* structure and the technological complexity of the executed processes. Depending on the assumed evaluation criteria the same system may be classified into two complexity types: class *P* or class *NP*. For realistic systems a formal definition of the system`s complexity is difficult and ambiguous. The system`s complexity may be expressed by a number being the result of the assumed evaluation algorithm, it may also be defined by a subjective grade in respect to the established criteria.

This article presents a complexity evaluation algorithm of cooperation for an exemplary company with a convergent manufacturing character. The formalized algorithm allows determining the level of cooperation complexity for any object. The formalized cooperation complexity is an absolute measure of [0,1] scale, accounting for the number of elements belonging to the company`s structure and the number of relations determined on *PS*.

2. CHARACTERISTICS OF THE DISCUSSED MANUFACTURING SYSTEM

Manufacturing systems are complex arrangements and dynamic [1], which according to the complex systems theory are subject to decomposition according to a set division criteria. Classification criteria selection depends on the division criteria. Performing a multi-staged decomposition process formalizes the hierarchical system, where on each division stage has a number of independently varied number of subsystems. Every separated subsystem alone may be considered as a system with characteristic labor parameters, the main goal of which is executing assigned tasks. Manufacturing system analyses on an operational level firstly encompass research of the manufacturing processes by productiveness, quality, execution time etc. This article discusses a complex system of convergent manufacturing characteristics. The complexity of the discussed system results from many technologically diversified processes being executed within the structure area of the company [1]. Furthermore the structure of manufactured final products is also a complex system composed of multiple subsystems where each subsystem possesses an individual number of details. Details are simple products, which are manufactured within the discussed facility in a significant amount. Due to large assortment variety and final product customization, the company`s structure has a diversified material flow streams which undergo a fusing process at various manufacturing stages [2, 3]. Determining the complexity level of cooperation has been formalized for a cell manufacturing structure, the schematics of which have been shown in **Figure 1**. It may be noted that each determined subsystem (manufacturing department) has a different number of manufacturing cells, machines, workstations and operators. Such presented *PS* production structure may be presented as [4, 5]:

$$PS = \{E, R, F\} \qquad (1)$$

where: E - an element of the manufacturing system - considered departments D_i for $i = 1,2,...,n$, R - set of reactions determined in the appointed departments, F - a set of functions formalized within the PS area.

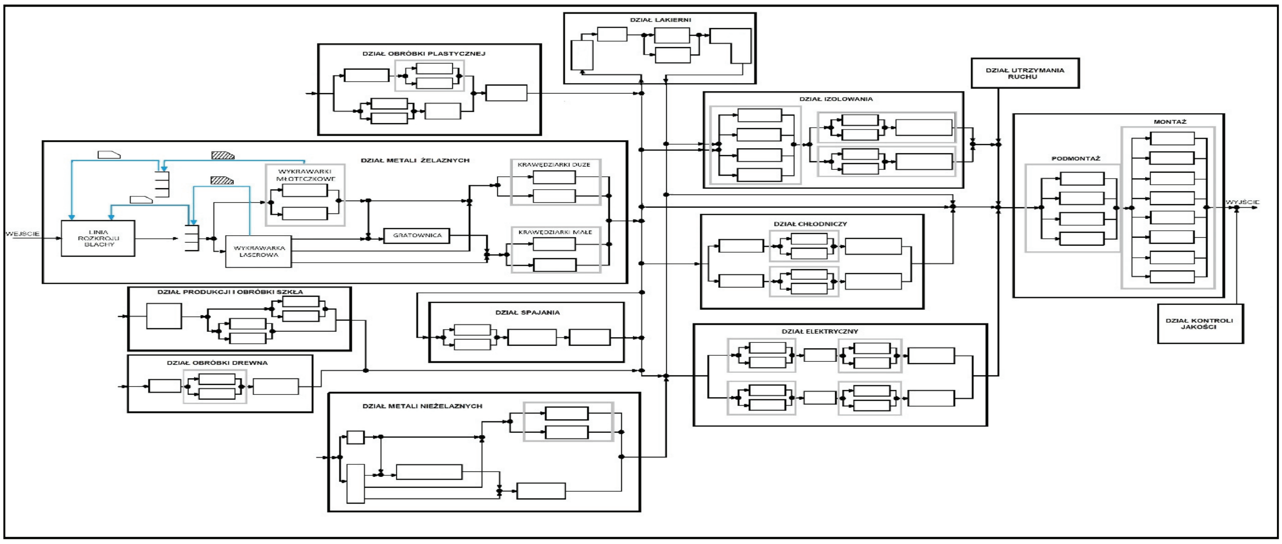

Figure 1 A schematic of the analyzed manufacturing structure

Relation is a strictly defined interaction between at least two elements of the set or between at least two divisible subsets. The most common defined relations within the manufacturing structures are the following relations:

- *symmetrical relations:*: $\forall x, y \in X : x R y \Rightarrow y R x$ - for any $X \subset PS$ subsystem elements, x is in a relation type y there is the same relation between y and x,
- *asymmetrical relations*: $\forall x, y \in X : x R y \wedge y \sim R x$ - for any $X \subset PS$ subsystem elements, x is in a determined relation with y there is no such relation between y and x.

The cooperation complexity algorithm has been defined in the criteria of dependency level of the defined subsystems, the structure of which can be seen in **Figure 1**. In this context the cooperation complexity of the manufacturing system PS, considered as a set of codependent departments accounts for the existing relation complexity level between subsystems (departments D_i) and their actual number of relations between concrete, defined D_i elements from the primary PS set.

2.1. Cooperation complexity defining algorithm

The formalized algorithm for determining cooperation complexity as one of the parameters for evaluating the total complexity of a system. The total complexity of a system is influenced by: the number of discussed subsystems, determining structural piety; the level and variety of implemented technical and technological solutions determining the functional complexity; and the number of independent flow streams of the processed materials, which is expressed by the value of the streams' complexity. To determine the inner structural cooperation complexity of a manufacturing system, i.e. α_C^{PS} complexity existing on the first level of decomposition, between the chosen subsystems - departments: D_i, D_j, where $i \neq j \wedge i, j \in \{1,2,...,n\}$, have been defined:

- set A all possible number of relations $R_{i,j}$, which differ between each other:

$$A = \{(i, j) : \forall (s, p) \in A \wedge (s, p) \neq (i, j) : R_{s,p} \neq R_{i,j}\} \tag{2}$$

where: $\# A \in \langle 1, n \rangle \cap N$

- $\beta_{i,j} \in (0,1]$ factor determining the level of relative dependency (relative relation complexity) between i - st. and j - st. department belonging to set $E \in PS$,

- $K_{i,j}$ set of all possible pairs (s,p), where $(s,p)\in\{1,2,\ldots,n\}$ - being a set of pairs of relation numbers between all departments creating $R_{s,p}$ relations identical to relations $R_{i,j}$:

$$K_{i,j}=\{(s,p):R_{s,p}=R_{i,j}\,;s,p,i,j\in\{1,2,\ldots,n\}\wedge s\neq p\wedge i\neq j\} \quad (3)$$

For sets of relations defined this way the cooperation complexity has been determined according to:

$$\alpha_C^{PS}=\sum_{\substack{(i,j)\in A\\ i\neq j}}(\beta_{i,j}\cdot k_{i,j}) \quad (4)$$

where: α_C^{PS} - the unconditional complexity factor of system PS cooperation determining the summary complexity of all possible $\beta_{i,j}$ complexities defined between two determined departments D_i and D_j; $\beta_{i,j}$ - relative complexity level factor of relation between D_i and D_j department; $k_{i,j}$ - determines the power of $K_{i,j}$ set ($k_{i,j}=\#K_{i,j}$), defines the number of set elements of all possible relation pair numbers between departments numbered (s,p) creating $R_{s,p}$ relations which are identical to relations $R_{i,j}$

The relative relation complexity level factor $\beta_{i,j}$ between D_i and D_j department, may be defined through a subjective evaluation relative to the established criteria. Then the evaluation result may be the following statement: *„relations between x and y are more complex than relations between v and z"*, when the subjectively assigned complexity indicator values fulfill the following condition: $\beta_{x,y}>\beta_{v,z}$. In order to determine credible criteria of complexity level evaluation, indicator $\beta_{i,j}$ may define for example the number and/or magnitude (mass or amount) of various flows through D_i and D_j department in a determined Δt timeframe. In that case indicator $\beta_{i,j}$ becomes the absolute evaluation of relation complexity between the discussed subsystems.

The cooperation complexity α_C^{PS} expressed by the formula (4) is defined by the existing number of relations between departments:: $1,2,\ldots,n$; where $R_{i,j}$ determine any relation between department D_i and D_j. For a reliability system defined this way it does not matter whether relations $R_{i,j}$ are symmetrical or asymmetrical, since there is one a single determined symmetrical relation or two differing asymmetrical relations between two established departments D_i and D_j:

- $R_{i,j}=R_{j,i}$ for symmetrical relations,
- $R_{i,j}\neq R_{j,i}$ for asymmetrical relations.

When all relation $R_{i,j}$ are different, then $k_{i,j}=1$ since a pair of $(i,j)\in K_{i,j}$. If there are several identical relations meeting the condition:

$$\exists\, s,p,i,j\in\{1,2,\ldots,n\};(s,p)\neq(i,j)\wedge s\neq p\wedge i\neq j\wedge R_{s,p}\neq R_{i,j} \quad (5)$$

then: $k_{i,j}\in(1;n(n-1))$ (6)

When all $R_{s,p}$ relations are the same and equal $R_{i,j}$, then

$$\exists\,(i,j)\in\{1,2,...,n\}\;\forall(s,p)\in\{1,2,...,n\};\,(s,p)=(i,j)\wedge s\neq p\wedge i\neq j\wedge R_{s,p}=R_{i,j} \tag{7}$$

then: $k_{i,j}=n(n-1)$ (8)

The value of the cooperation complexity factor is an absolute measure determined in the: $\alpha_C^{PS}\in[0,\infty)$ range.

The cooperation complexity of α_C^{PS} is expressed by formula (4) shaped by the relation complexity level $\beta_{i,j}$ and it`s actual number $k_{i,j}$. Cooperation complexity may also be defined by accounting for the influence of factor ξ expressing the relation of the total number of pairs of various relations taking place between any department to the total number of relations between departments in the system. α_{C*}^{PS} therefore is defined accordingly:

$$\alpha_{C*}^{PS}=(1+\xi)\sum_{\substack{(i,j)\in A\\ i\neq j}}\left(\beta_{i,j}\cdot k_{i,j}\right) \tag{9}$$

where: $\xi=\dfrac{\#A}{n(n-1)}$ (10)

where: ξ - relation of the actual number of various relations in relation to the total number of definable relations in the considered subsystems - departments D_i and D_j of $E\in PS$ set; value $\xi\in(0,1]$.

The cooperation complexity α_{C*}^{PS} defined by formula (9) considers complexity flow α_C^{PS} from formula (4) and the product of α_C^{PS} and factor ξ. In a dynamically changing system, increasing the value of various relations in regards to all those possible, causes an increased complexity value of the entire system. For this reason with a large variety of relations determined within the PS structure, indicator $\xi\to 1$. It is therefore possible for a system to exist, which is comprised of multiple elements connected by various relations, being more complex than one that has multiple elements (subsystems), but have only a single type of relations identified.

2.2. Analysis of the algorithm`s correctness

Formula (9) determining the cooperation complexity value for a complex system, accounts for the number of selected subsystems, the number of relations identified in the entire system and the complexity level of said relations. An analysis of the correctness of the formulated algorithm for determining the value of cooperation complexity relies on checking two variants: 1. Does value α_{C*}^{PS} increase along with the rise of the number of selected subsystems? and 2. Does the value of α_{C*}^{PS} rise along with the increase of the number of relations with a constant number of selected subsystems?

Variant1: the considered scenario is one where each system (S_I and S_{II}) has a certain number of relations of only one type determined.

$$\alpha_{C*}^{S_I}=\left(1+\frac{1}{n(n-1)}\right)\cdot\beta_{i,j}\cdot n=\frac{n(n-1)+1}{n-1}\cdot\beta_{i,j} \tag{11}$$

$$\alpha_{C*}^{S_{II}}=\left(1+\frac{1}{n(n-1)}\right)\cdot\beta_{i,j}\cdot(n+1)=\frac{n(n+1)+1}{n}\cdot\beta_{i,j} \tag{12}$$

I may confirm the correctness of formula (9) by verifying whether for any $n\geq 2$ and $n\in N$ condition $\alpha_{C*}^{S_I}<\alpha_{C*}^{S_{II}}$ is true.

Therefore:

$$\frac{n(n-1)+1}{n-1}\cdot\beta_{i,j} < \frac{n(n+1)+1}{n}\cdot\beta_{i,j} \quad (13)$$

After transformations we obtain: $n^2(n-1)+n < n(n+1)(n-1)+(n-1)$ (14)

Finally the following condition is tested: $n^2-n-1>0$ where $\Delta=5$ (15)

therefore $n_1=\frac{1-\sqrt{\Delta}}{2}\approx -0{,}6$ and $n_2=\frac{1+\sqrt{\Delta}}{2}\approx 1{,}6$ (16)

so $n\in(1,,6;\infty)\cap N$ (17)

The case analyzed above concludes, that along with the increase of structural complexity (i.e. increase in the number of subsystems) of the analyzed object, the value of cooperation complexity increases, if there is at least one new cooperation relation between the newly isolated object. An increase in the cooperation complexity value indicator also takes place if the number of relations on the determined structure PS increases. Adequately to the decrease of the number of elements or the decrease of the number of relations there is a decline of the complexity cooperation indicator value.

Varian 2: checking whether the defined formula (9) determining the cooperation complexity value is correct when the number of relations on a system with unchangeable number of separated subsystems increases. To demonstrate the correctness of the formulated formula, two systems S_I and S_{II} have been compared, which both have an $n\geq 2$ number of selected subsystems with a number of defined subsystems where the number of relations defined for systems S_I and S_{II} equals accordingly l for system S_I and $l+1$ for system S_{II}.

The assumptions for variant 2: for systems S_I and S_{II} each defined relation is different, i.e.: value $k_{i,j}=1$ in formula (9), furthermore for system S_I: $l<n(n-1)$ and for system S_I: $l\leq n(n-1)$.

Confirmation of correctness was done through a verification of the dependencies: $\alpha_{C*}^{S_I}<\alpha_{C*}^{S_{II}}$ where:

$$\alpha_{C*}^{S_I}=\left(1+\frac{l}{n(n-1)}\right)\sum_{i=1}^{l}\beta_i=\frac{n(n-1)+l}{n(n-1)}\sum_{i=1}^{l}\beta_i \quad (19)$$

$$\alpha_{C*}^{S_{II}}=\frac{n(n-1)+(l+1)}{n(n-1)}\sum_{i=1}^{(l+1)}\beta_i \quad (20)$$

for: $l\in\{1,2,...,n(n-1)-1\}\wedge n\geq 2$, $n\in N$ (18)

After substituting the following was obtained

$$\frac{n(n-1)+l}{n(n-1)}\sum_{i=1}^{l}\beta_i<\frac{n(n-1)+(l+1)}{n(n-1)}\sum_{i=1}^{(l+1)}\beta_i \quad (21)$$

After transformation the following was obtained:

$$[n(n-1)+l]\sum_{i=1}^{l}\beta_i<[n(n-1)+(l+1)]\sum_{i=1}^{l}\beta_i+[n(n-1)+l+1]\beta_{l+1} \quad (22)$$

Final result: $0<\sum_{i=1}^{l}\beta_i+[n(n-1)+(l+1)]\beta_{l+1}$ (23)

Analysis of the above variant suggests that increasing the number of relations by 1 for a system with the same number of selected subsystems, increases the value of cooperation complexity indicator by $\sum_{i=1}^{l}\beta_i+[n(n-1)+(l+1)]\beta_{l+1}$. Hence a system the area of which has a certain number of relations defined, the next relation different than others is defined, then the system`s complexity value increases relatively to all existing relations (part of formula (23): $\sum_{i=1}^{l}\beta_i$) and to the relation additionally defined (added) i.e.: part of formula (23): $[n(n-1)+(l+1)]\beta_{l+1}$.

Formula (9) is formulated correctly as both the change (increase or decrease) in the number of departments with a constant number of existing relations and the change in the number of relations with a constant number of departments causes an adequate change (an increase or decrease) of the indicator α_{C*}^{PS} value

3. TOTAL SYSTEM COMPLEXITY

System complexity is a trait which influences the difficulty of being defined, difficulty of understanding, difficulty of use - generally the difficulty is in comprehension, application and management [2,6]. The complexity of a manufacturing system includes the following aspects [7]: complexity of the system`s structure, number of elements, the number and level of relations existing between the system`s elements, the implemented technological solutions, realized functions, etc.. Total manufacturing system complexity should therefore be defined by an aggregated measure of the system`s complexity, evaluated multidimensionally. A coherent evaluation in a comprehensive take should account for a multi-criteria evaluation of many areas of action. In order to determine the total complexity of a system the following complexity areas must be defined:

- the system`s structural complexity $\alpha_S^{PS}$, which defines the system`s trait unconditionally dependent of inter-structural relations in the selected SP elements - selected subsystems, departments,
- the manufacturing system`s functional complexity α_F^{PS} interpreted as a synonym of usefulness, determining the possibility of realizing the functionality function, which means achieving the assumed goal,
- flow stream complexity α_R^{PS}, defined by the number of possible full, independent manufacturing paths,
- cooperation complexity α_C^{PS}.

Due to this the defined singular system complexity indicators, which are a measure of a single chosen area, should constitute part of the entire object`s evaluation. The multi-criteria algorithm for evaluating manufacturing system complexity should be determined in such a way, so that no singular parameter value dominates the others. Then the total complexity of a manufacturing system α_T^{PS} accounting for and adequately proportional share of the structural complexity α_S^{PS}, functional complexity α_F^{PS}, cooperation complexity α_C^{PS} and flow complexity α_R^{PS}, has been determined as follows:

$$\alpha_T^{PS}=W(\alpha_S^{PS})\cdot\alpha_S^{PS}+W(\alpha_F^{PS})\cdot\alpha_F^{PS}+W(\alpha_C^{PS})\cdot\alpha_C^{PS}+W(\alpha_R^{PS})\cdot\alpha_R^{PS} \tag{24}$$

where: $W(\alpha_{XX}^{PS})$ - the weight share of indicator α_{XX}^{PS} for indicators: α_S^{PS}, α_F^{PS}, α_C^{PS} and α_R^{PS}.

In order for condition (24) determining the manufacturing system`s total complexity accounting for all complexity areas, which has been defined basing on the weight share of individual indicator values, the following dependency must be fulfilled: $W(\alpha_S^{PS})+W(\alpha_F^{PS})+W(\alpha_C^{PS})+W(\alpha_R^{PS})=1$. Expressed by formula (24) total PS complexity is a defined, multi-criteria algorithm for evaluating a complex manufacturing structure.

4. CONCLUSION

A complex analysis of realistic manufacturing objects is often a difficult task, with the difficulty level rising proportionally as the manufacturing systems complexity increases. Many publications consider any system to be complex if it meets the criteria for complex systems, i.e.: it has a relatively large amount of elements and/or the relations identified between chosen elements display a relatively high level of complexity. This kind of manufacturing systems complexity formalization is a subjective evaluation, not accounting for objective parameters of the evaluation. This article presents the algorithm for determining the cooperation complexity factor value for a complex manufacturing system. Applying the presented algorithm allows for an objective determination of cooperation complexity value of any system. Furthermore, if the same system decomposition criteria into sets of subsystems and subsequently cooperation complexity values are determined according to the presented algorithm for two different systems, it will be possible to recognize which of the two compared systems is the “most” complex.

This article distinguishes three areas. The first part considers formulating the algorithm for determining cooperation complexity value. The second part is verifying the correctness of the presented algorithm for extreme cases. The closing part of this article points out that while defining and formulating the complexity of manufacturing systems, four co-dependent areas of system operation must be considered. And finally the third part presents the formula for determining the total system complexity accounting for the weight share of individual complexity of a given area of operation.

REFERENCES

[1] DONNER, Reik V., SCHOLZ-REITER, Bernd, and HINRICHS, Uwe. Nonlinear characterization of the performance of production and logistics Networks. *Journal Of Manufacturing Systems*. 2008. vol. 27, no. 2, pp. 84-99.

[2] EFTHYMIOU, Konstantinos, PAGOROPOULOS, Aris, PAPAKOSTAS, Nikolaos, MOURTZIS, Dimitris, and CHRYSSOLOURIS, George. Manufacturing systems complexity: an assessment of manufacturing performence indicators unpredictability. *CIRP Journal of Manufacturing Science & Technology*. 2014. vol. 7, no. 4, pp. 324-334.

[3] FAN, Guoliang, LI, Aiping, MORONI, Giovanni, XU, Liyun and LIU, Xuemei. Operation-based Configuration Complexity Measurement for Manufacturing System. *Procedia CIRP*. 2017. vol. 63, pp. 645-650.

[4] KLIR, George J. Complexity: some general observations. *Systems Research*. 1985. vol. 2, no. 2, pp. 131-140.

[5] MESAROVIC, Mihajlo D., MACKO, D. and TAKAHARA, Yasuhiko. *Theory of Hierarchical, Multilevel, Systems*. Series: Mathematics in science and engineering: vol. 68. New York, Academic Press. 1970.

[6] MYRODIA, Anna. *Complexity Management - A multiple case study analysis on control and reduction of complexity costs*. Publisher’s PDF. Denmark 2016.

[7] SAMY, S. N. and ELMARAGHY, Hoda . A model for measuring complexity of autometed and hybrid assembly systems. *The International Journal of Advanced Manufacturing Technology*. 2012. vol. 62, no. 5, pp. 813-833.