

## DETECTING THE TRANSPORT HUBS PROBLEM: AN EVOLUTIONARY APPROACH

MAŻBIC-KULMA Barbara<sup>1</sup>, OWSIŃSKI Jan W.<sup>2</sup>, STAŃCZAK Jarosław<sup>1,2</sup>, BARSKI Aleksy<sup>2</sup>,  
SĘP Krzysztof<sup>1,2</sup>

<sup>1</sup>Wyższa Szkoła Informatyki Stosowanej i Zarządzania, Warszawa, Poland, EU

<sup>2</sup>Instytut Badań Systemowych PAN, Warszawa, Poland, EU

### Abstract

In this paper we consider the well known Hub and Spoke problem according to the Warsaw Public Transport System. Our approach is based on data available on the website of the ZTM (Zakłady Transportu Miejskiego - Urban Transport Company) and an evolutionary algorithm method that detects logistic hubs well connected (with the high capacity of available transport means) with the center of the city and other detected hubs. These hubs can become the skeleton of the public transport system and for instance can be good points for localization of the Park and Ride car parks.

**Keywords:** Hub and spoke, evolutionary algorithm

### 1. INTRODUCTION

The dynamic development of the agglomerations causes several new challenges. The continuous increasing of pollutions and expanding the traffic as a reason of growing traffic jams make the popularization of the public transport one of the most urgent task. One of the essential problems for this task is the improvement of the travel time. Generally, the concept behind such a system is simple: let the public transport be effective as possible, making using public transport cheaper than the others. The widely used in transportation systems idea of hub and spoke (H&S) structure can be helpful also in the case of the urban transport system, where fast transport means like metro or urban trains can create a skeleton of fast and high capacity connections and slower transport means like trams or buses can support local connections. In the second part of the XX century the H&S structure began to gain in popularity especially after the deregulation in 1978 in the US airline [1]. Since 1986 the hub and spoke problem has become a distinct research area [2].

To reorganize the transportation system into the instance of H&S structure it is necessary to detect potential hubs in the graph of the city logistic system. This quite a big computational task is a job for evolutionary method often used for solving of this type problems. In this paper we present our method useful for obtaining H&S structure in the logistic system.

### 2. GRAPHS AND THEIR RELEVANT STRUCTURES

Since we shall be modeling the entirety of the hub and spoke problem through the graph representation and then solving it as a problem defined on graphs, we start with the basic notions from graph theory, here given following [16].

A **graph** is a pair  $G = (V, E)$ , where  $V$  is a non-empty set of *vertices* and  $E$  is a set of *edges*. Each edge is a pair of vertices  $\{v_1, v_2\}$  with  $v_1 \neq v_2$ . A **degree** of a vertex is the number of edges to which this vertex belongs. A **clique** (a *complete subgraph*)  $Q = (V_q, E_q)$  in a graph  $G = (V, E)$  is a graph such that  $V_q \subseteq V$  and  $E_q \subseteq E$  and  $Card(V_q) = 1$  or each pair of vertices  $v_1, v_2 \in V_q$  fulfils the condition  $\{v_1, v_2\} \in E_q$  [4]. Each subgraph of a clique is a clique. An  **$\alpha$ -clique** [6], [11], [12], [14] can be defined as follows: let  $A=(V', E')$  be a subgraph of graph  $G = (V, E)$ ,  $V' \subseteq V$ ,  $E' \subseteq E$ ,  $k = Card(V')$  and let  $k_i$  be a number of vertices  $v_j \in V'$  that  $\{v_i, v_j\} \in E'$ .

1. For  $k = 1$  the subgraph  $A$  of graph  $G$  is an  $\alpha$ -clique( $\alpha$ ).
2. For  $k > 1$  the subgraph  $A$  of graph  $G$  is an  $\alpha$ -clique( $\alpha$ ) if for all vertices  $v_i \in V'$  fulfill the condition  $\alpha = (k_i + 1) / k$ , where  $\alpha \in (0, 1]$ .

Further on we will use the notion of  $\alpha$ -clique in the sense of  $\alpha$ -clique( $\alpha$ ) for an earlier established  $\alpha$ . A subgraph of an  $\alpha$ -clique may not be an  $\alpha$ -clique for the established  $\alpha$ .

A **hub and spoke** structure (Figure 1b) is a graph  $H_s = (G_h \cup G_s, E)$  where the subset  $G_h$  corresponds to at least a connected graph (of hubs) with the relevant subset of set  $E$ , each vertex of subset  $G_s$  (of spokes) has degree 1 and is connected exactly with one vertex from subset  $G_h$  [6], [7].

The proposed method uses a predetermined by some expert number of communication hubs, with the possibility of directly determining which nodes should become hubs or selecting them by the solving method.

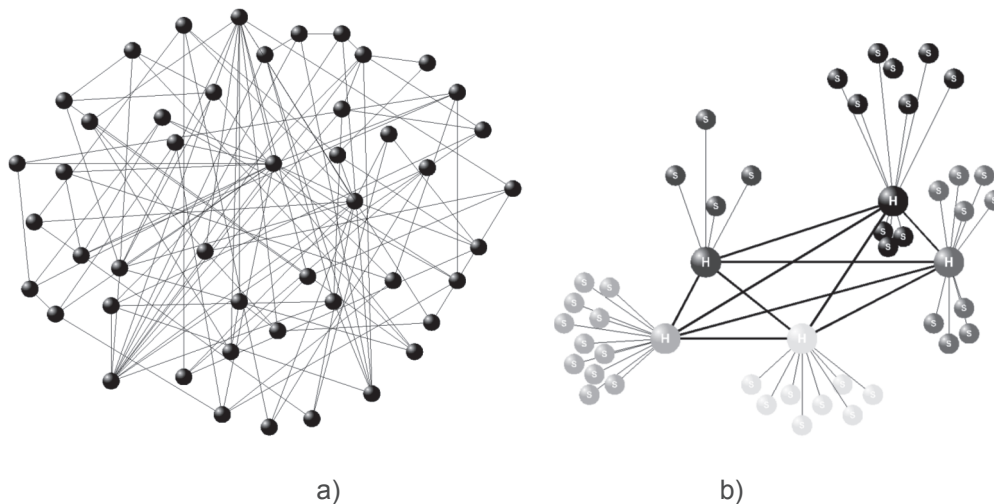


Figure 1 a) a source graph and b) the hub and spoke structure obtained

### 3. THE PROBLEM REPRESENTATION

The structures we introduced in the previous section are helpful in solving the hub and spoke problem, or they can even directly represent such solutions, since edges correspond to transport-wise connections, and nodes represent transport nodes. Thus, by finding these structures, we obtain the solution or the basis for determining the solution.

We assume there is a direct connection between two communication graph nodes if there is a public transport line they are belonging to. Thus, the graph describing the transportation network, used in our approach, is not a simple graph representation of city transportation routes, but a denser structure with interconnections among stops belonging to one transport line. Different lines may have common stops, so the transportation graph consists of overlapping “blocks” of lines. In our approach the graph of connections is a weighted graph, where all edges may have different values assigned. These values may represent not only the fact of connection existence but also some parameters of it: the number of scheduled courses over the connection (frequency), the travel time, potential capacity of the given means of transport (within a given period of time), actual/estimated number of passengers (in the vehicle: descending / boarding).

In this paper we use values (weights) assigned to edges proportional to the connection capacity. For stops, common for a few or more transport lines, these values are properly modified, for instance the capacity and frequency can be added, the travel time averaged, as so to consider more transport means available. The hub and spoke method in the big city case can assure fast and high capacity connections (in real cases the fastest

means of transport are: metro, urban trains and fast trams) among hubs and to the center of the city, and slower or smaller capacity ones to less important stops - spokes (in this case buses and ordinary trams). Such transport organization in the city can significantly improve the transport system efficiency [9], [10].

#### 4. THE EVOLUTIONARY METHOD TO OBTAIN THE HUB AND SPOKE STRUCTURE

Evolutionary algorithms (EAs) are often used to solve hard graph problems such as graph coloring, TSP, graph partitioning, maximum clique problem, etc. [3], [5], [15], [17], thus, it is justified to use the evolutionary algorithm in the presented graph transformation problem. Basic EA method is a well-known technique, but this scheme requires problem specific improvements to work efficiently [8]. The adjustment of the evolutionary algorithm to solve a problem requires proper encoding of solutions, development of specialized genetic operators proper for the analyzed data structure and the solved problem and, finally, the fitness function (or another manner of evaluating solutions) to be optimized by the algorithm.

The selected *hubs* should constitute a complete graph, but when the connections between nodes are very sparse or are determined as existing junction nodes (for instance: railway stations), it is admitted that the subgraph of hubs constitute an  $\alpha$ -*clique* with  $\alpha$  as high as possible or a connected graph (when the whole graph of connections is very sparse). Considering a weighted graph, weights of edges connecting hubs should have values as big as possible.

A member of the population contains several data items, including: vectors of detected hubs and their spokes, a vector of real numbers, describing its knowledge of genetic operators and the index of the operator chosen to modify the solution in the current iteration (for details see [13]).

The quality function to obtain the desired *hub and spoke* structure with the predetermined number of kernel nodes is presented in formula (1):

$$\max Q = \sum_{i=1}^n \left( \sum_{j=1}^m w_{ij}^C + \sum_{l=i+1}^n w_{il} + \sum_{p=1}^k w_{ip} \right) \quad (1)$$

$n$  - imposed number of hubs,  $k$  - number of nodes (stops) in the considered graph,  $w_{ij}^C$  - weight of the connection between  $i^{\text{th}}$  hub and  $j^{\text{th}}$  stop of the set of  $m$  important stops (we decided to emphasize several important communication stops in the city as a virtual aim of the majority of commuters) in the center of the city,  $w_{il}$  - weight of the connection between  $i^{\text{th}}$  hub and hub  $l^{\text{th}}$ ,  $w_{ip}$  - weight of the connection between  $i^{\text{th}}$  hub and  $p^{\text{th}}$  node of the graph.

The quality function (1) maximizes connections among: selected hubs (  $\sum_{l=i+1}^n w_{il}$  ), hubs and the set of the most

important virtual aims of the commuters' travels (  $\sum_{j=1}^m w_{ij}^C$  ) and each hub and remaining stops (  $\sum_{p=1}^k w_{ip}$  ). The

connection weights in the graph represent a connection capacity, means as a number of average transportation vehicles in one hour.

This problem can be solved using a set of the specialized genetic operators, modifying the structure of the population member. When the executed operator tries to move one node (spoke) to the shell associated with another hub, must the first to be checked for the connection with this new hub. If there is not, the operation is canceled and no modifications are performed, so as to preserve the feasibility of the solution. The set of genetic operators consists of: mutation - exchange of randomly chosen nodes in different sets of spokes, relocation of a randomly chosen node to a different set of spokes, exchange of a randomly selected hub for randomly selected spoke, this operator is inactive when kernel nodes are explicitly assigned.

## 5. RESULTS OF THE COMPUTER SIMULATIONS

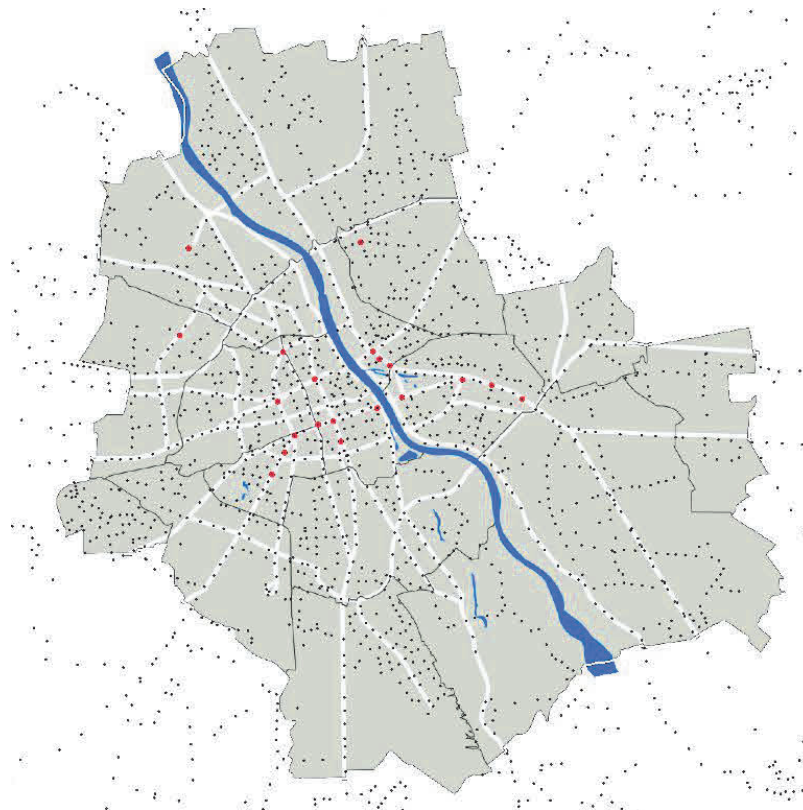
### 5.1. The real data of the Warsaw transport system

The real data describing the Warsaw transportation network were obtained from the ZTM website [18]. As it can be seen in subsequent figures, the network is well developed, and composed of: subway - 2 lines, fast city trains (SKM) / suburban trains (WKD) / trains (KM) - 13 lines, trams - 25 lines, buses urban / suburban / night - 199 lines.

We take into account almost all means of public transport, only some private bus lines, taxis, long distance buses and trains are not considered. Generally it is very difficult to deal with the real data describing the city transport system, because they change almost every day, there are often flaws in the data and unexpected variants of some communication lines circulation (one-direction lines, different routes in different hours, loops, etc.). The whole transport network has about 10 000 (physical) stops but for computational purposes this value can be reduced to about 2 500 integrated nodes. An integrated node consists of corresponding stops in both directions and often also several sub-stops in both directions in places with very intensive commuter traffic.

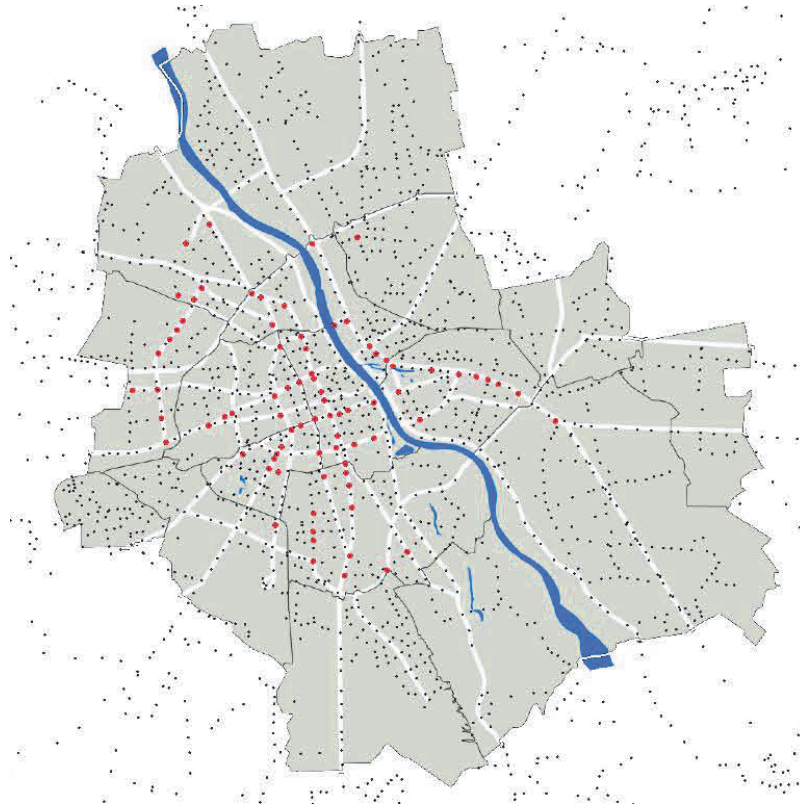
### 5.2. Obtained results

The figures in this section presents candidates for hubs detected by evolutionary method on the basis of the capacity of their connections with the virtual center of the city and with other detected hubs. The simulations were performed for 20, 50 and 100 imposed hub numbers.

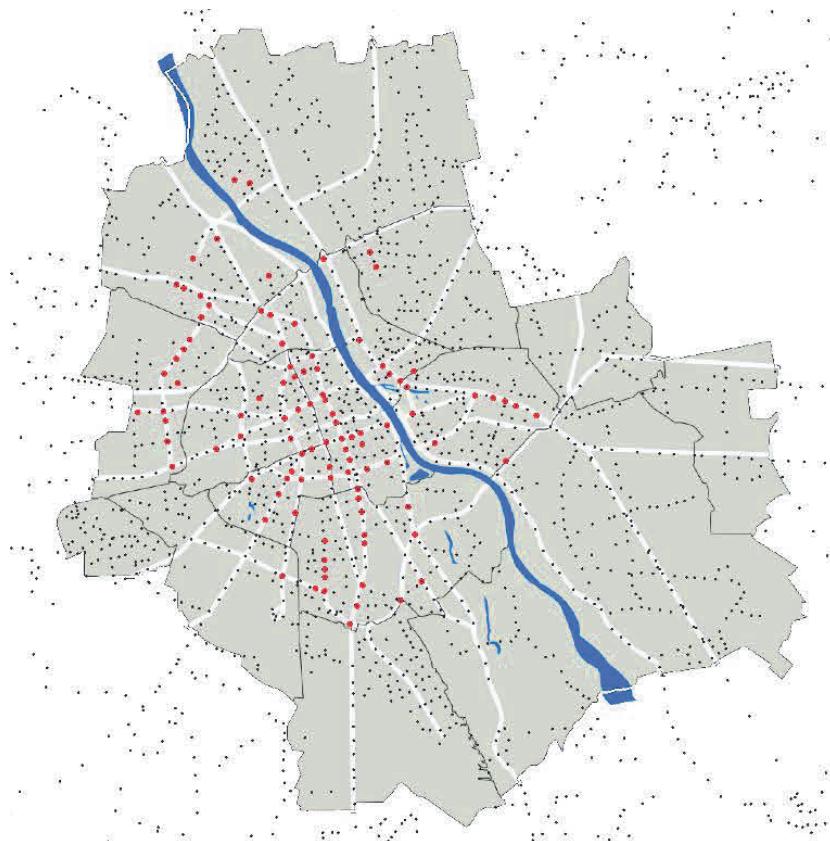


**Figure 2** The scheme of Warsaw, small dots - ordinary stops, 20 big dots - candidates for hubs

As we can see the method proposes several candidates located along the most important communication arteries in the city. It is obvious that only several of them can be turned into communication hubs, but this could definitely improve the transport system in Warsaw, improving the life level of residents of the capital of Poland. It should be noticed that detected hub candidates are mainly connected with rail transport means.



**Figure 3** The scheme of Warsaw, small dots - ordinary stops, 50 big dots - candidates for hubs



**Figure 4** The scheme of Warsaw, small dots - ordinary stops, 100 big dots - candidates for hubs

## 6. CONCLUSION

The approach presented appears as a promising methodology for supporting the planning and improving the public transport model and also other logistic branches, where the hub and spoke methodology can be applied without big costs connected to the construction of new communication lines.

## REFERENCES

- [1] BAILEY E. E. Airline Deregulation Confronting the Paradoxes, Regulation, The Cato Review of Business and Government 15, No. 3, 1992.
- [2] CAMPBELL J.F., O'KELLY M.E. Twenty-Five Years of Hub Location Research, Transportation Science, 2012, pp. 153-169.
- [3] CHEN Z.Q., WANG R.L., OKAZAKI K. An Efficient Genetic Algorithm Based Approach for the Minimum Graph Bisection Problem, IJCSNS International Journal of Computer Science and Network Security, Vol. 8 No. 6, 2008, pp. 118- 24.
- [4] CORMEN T.H., LEISERSON C.E., RIVEST R.L., STEIN C. Introduction to algorithms, MIT, 2009.
- [5] MARCHIORI E. A Simple Heuristic Based Genetic Algorithm for the Maximum Clique Problem, Proceedings of the 1998 ACM symposium on Applied Computing, 1998, pp. 366-373.
- [6] MAŹBIC-KULMA B., POTRZEBOWSKI H., STAŃCZAK J., SĘP K. Evolutionary approach to solve hub-and-spoke problem using  $\alpha$ -cliques, Evolutionary Computation and Global Optimization, Prace naukowe PW, Warszawa, 2008, pp. 121-130.
- [7] MAŹBIC-KULMA B, POTRZEBOWSKI H., STAŃCZAK J., SĘP K. Evolutionary approach to find kernel and shell structure of a connection graph, Total Logistic Management, AGH, Cracow, 2009, pp. 37-50.
- [8] MICHALEWICZ Z. Genetic Algorithms + Data Structures = Evolution Programs, Springer Verlag, Berlin-Heidelberg, 1996.
- [9] O'KELLY M.E., BRYAN D. Interfacility interaction in models of hubs and spoke networks. Journal of Regional Science, 42 (1), 2002, pp. 145-165.
- [10] O'KELLY M.E. A quadratic integer program for the location of interacting hub facilities, European Journal of Operational Research 32, 1987, pp. 392-404.
- [11] POTRZEBOWSKI H., STAŃCZAK J., SĘP K. Evolutionary Algorithm to find graph covering subsets using  $\alpha$ -cliques, Evolutionary Computation and Global Optimization, Prace naukowe PW, Warszawa, 2006, pp. 351-358.
- [12] POTRZEBOWSKI H., STAŃCZAK J., SĘP K. Separable decomposition of graph using alpha-cliques, in: Kurzyński. M., Puchała E., Woźniak M, Żołnierek A. (Eds.): Computer Recognition Systems 2, in: Advances in Soft Computing, Springer-Verlag, Berlin-Heidelberg, 2007, pp. 386-393.
- [13] STAŃCZAK J. Biologically inspired methods for control of evolutionary algorithms, Control and Cybernetics, 32(2), 2003, pp. 411-433.
- [14] STAŃCZAK J., POTRZEBOWSKI H., SĘP K. Evolutionary approach to obtain graph covering by densely connected subgraphs, CONTROL AND CYBERNETICS, vol. 41, No. 3, 2011, pp. 80-107.
- [15] TALBI E.G. BESSIERE P. A parallel genetic algorithm for the graph partitioning problem, Proceedings of the 5<sup>th</sup> International Conference on Supercomputing, ACM, New York, 1991, pp. 312-320.
- [16] WILSON R.J. Introduction to Graph Theory, Addison Wesley Longman, 1996.
- [17] YU T.L., GOLDBERG D.E., YASSINE A., YASSINE C.A. Genetic algorithm design inspired by organizational theory, Genetic and Evolutionary Computation Conference Chicago, Illinois, USA, Springer-Verlag, Heidelberg, Lecture Notes in Computer Science, Vol. 2724, 2003.
- [18] [www.ztm.pl](http://www.ztm.pl) and <ftp://rozklady.ztm.waw.pl/>