

COMBINED USE OF SIMULATION AND OPTIMIZATION MODELS AS A DECISION SUPPORT TOOL FOR ROBUST INVENTORY ROUTING PROBLEMS

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Abstract

The combination of decisions regarding inventory management and vehicle routing decision leads to complex combinatorial optimization problem called the Inventory Routing Problem (IRP). Several heuristic algorithms for solving IRP were proposed in recent years. A lot of them are based on static and deterministic versions of IRP. Based on our management experience we identified a lot of situations where the VMI approach is used but the dynamic version of the IRP has to be considered. Such situations occur in case of LPG supplies to fuel stations. There are some proposals of heuristic policies which can be used in above mentioned circumstances. In our proposal after each delivery the optimization models are applied in order to optimize routes and delivery dates. In our paper we presented the combined use of optimization models and discrete event simulation to evaluate the influence of different variability of selected models' parameters on the final plan. Simulation models have proved very useful as an aid to build dynamic and robust plans for considered inventory routing problem. We presented results of extensive simulation analysis of randomly generated cases for comparison our proposals and some policy solutions.

Keywords: VMI, VRP, IRP, DSS

1. INTRODUCTION

The fuel stations supply problem belongs to the major transportation and logistics problems with very strong economic impact. Autogas deliveries to fuel stations is a particular case of this problem. As a repetitive distribution of a homogenous product by a homogenous fleet of tank trucks it can a priori be considered as a very good research area for IRP solutions. Deliveries of petrochemical products to the fuel stations in most European countries are performed by highly specialized hauliers under centralized planning and scheduling conditions. The successful implementation of VMI (Vendor Managed Inventory) strategy can lead to high productivity results, high customer service level and efficient organization. One of the key factors for this approach is the adequate DSS (Decision Support System) for deliveries planning and scheduling. This paper focus on some logistic aspects of this complex and important problem basing on real-life data from the transportation company and develops some specific methods in order to increase the competitiveness of transportation operations.

The research method presented is an attempt to use time-discretized integer programming models to solve a real life IRP. The proposed formulation is based on location based heuristics (LBH) introduced by Bramel and Simchi-Levi [1]. In comparison to the primary formulation in this approach more than one item was considered. Additionally, based on empirical observations the seed sets technique was simplified and the decision model was slightly changed. The considered algorithm was also implemented by Hanczar [2] to solve the fuel distribution problem.

Until recently, optimization and simulation were kept pretty much separate in practice, even though there was a large body of research literature relevant to combining them. In the last two decades, however, optimization routines have prominently worked their way into and with simulation packages (Fu [3]). The combined use of optimizing methods of mathematical models (MIP) with discrete event simulation models can contribute to the formation of more accurate and more robust plans. Developed simulation models will allow to study of the impact of demand variability on distribution results.

2. SCOPE OF RESEARCH

The specificity of the transport services market of petroleum products and autogas in Poland indicates the possibility of the use of decision support models (in particular IRP approach) for delivery planning. The main key factors are (1) product homogeneity (in case of autogas / LPG we're dealing in fact with only one product, in the case of fuels with a maximum of 6 products); (2) existing infrastructure including telemetry systems that provide information about the inventory level of gas and fuels at service stations several times a day; (3) increasing frequency of application of Haulier Managed Inventory (HMI) approach by fuel suppliers, according to which the transport company is responsible for ensuring the availability of fuel at the station. On the other hand, important variations of daily demand on fuel stations is a significant factor to be considered for application of IRP models. In vast majority of theoretical papers concerning IRP, demand is considered deterministic even within the whole planning horizon.

The authors of the present study decided to implement the following research tasks. It was assessed in the first step how advantages of IRP application for fuel distribution can be affected in real life by fuel stations demand variability. In the first stage it was verified whether the variability of real data of consumption allows the use of IRP at all. In the second stage it was verified for which patterns of demand variability IRP models can be applied and the length of the planning horizon was determined.

Implementation of the second stage was related to proposal of a method of solving the present problem of fuel supplies that would be robust against high demand variability. Although the literature on the subject begin to provide some proposals concerning these issues, they are based in most cases on the use of simple supply rules heuristics or forecasts application. Unfortunately, none of these approaches apply for the case of Polish LPG deliveries market where consumption variability arrive to 40% or even more. The proposal provided by the authors of this paper, which in such conditions can significantly improve the performance of algorithms used, consists in launching the route optimization executed after each delivery. Combined use of the optimization approach (for IRP solutions) and simulation approach (to verify the effects of planning in stochastic conditions) allowed to point out the advantages of such an approach.

3. LITERATURE REVIEW

The paper by Beltrami and Bodin [4] may be deemed a pioneering work regarding IRP problems. This work focused on modeling and simple solution techniques. In the following papers by Fisher et al. [5] and Bell et al. [6] mixed integer programming was used first to obtain a solution for the IRP instance. Subsequently, the first approach to solve a large IRP instance was made by Golden et. al. [7] and by Dror [8], they investigated the large distribution system of liquid propane to residential and industrial customers. Burns et al. [9] try an analytic approach to solving IRP. They considered the optimal trade-off between inventory and transportation costs. The two distribution strategies are taken into account: direct shipping (i.e. shipping separate loads to each customer) and peddling (i.e., dispatching trucks that deliver items to more than one customer per load). The presented results indicate that the cost trade-off in each strategy depends on shipment size. For direct shipping, the optimal shipment size is given by the economic order quantity (EOQ) model, while for peddling; the optimal shipment size is a full truck. In the latter case, trade-off also depends on the number of customers included on a peddling route.

Parallel to the deterministic IRP the large area of research where the stochastic version of the IRP is considered. This field started with papers by Dror and Trudeau [10] and Dror Laporte and Trudeau [11], where authors consider the VRP with stochastic demand. However, due to the fact that in this paper stochastic formulation is not considered, a more detailed review of the paper has been omitted. A complete review of methods and algorithms for solving stochastic IRP problems may be found in Schwarz et al. [12] or Cordeau et al. [13]. Concerning simulation and optimization combined approach, one of the recent reviews that can be considered is Figueira and Almada-Lobo [14].

4. METHODOLOGY OF COMBINING SIMULATION AND OPTIMIZATION MODELS FOR THE ROUTES AND DELIVERY DATES

In our approach we consider combined use of simulation and optimization models for robust solving of inventory routing problem of LPG deliveries to petrol stations within one region in Poland (one depot, multiple vehicles, several dozen customers). Starting from the optimisation model in order to determine an optimal or suboptimal MIIRP solution of the problem, our approach proceeds with application of discrete event simulation tool (DES), which is able to provide information about whole system behavior and its reactions to LPG demand variations at petrol stations (see **Figure 1**). The solution generated by optimization model is used as input for simulation model to verify the feasibility and robustness of the computed solution through the generation of different scenarios which consider different levels of demand variability typical to real life systems.

The results of the simulation experiments, allowing an evaluation of the system performance, can support the detection of the current solution weaknesses and limitations of the initial problem MIIRP solution. The feedback loop is then realised going back to the optimisation phase with the new information generated by the simulation model. This information is used to improve the initial optimal or suboptimal solution. This approach allows to evaluate the relevant system performance in case of different levels of demand variations and indicate whether re-planning during the day is necessary and possible.

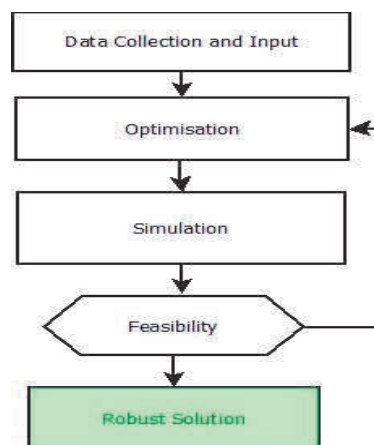


Figure 1 Overview schema of simulation-optimization integrated approach

In order to describe the demand variability at the stations we apply coefficient of variation (CV) which is defined as the ratio of the standard deviation σ to the mean μ :

$$CV = \frac{\sigma}{\mu} \tag{1}$$

It shows the extent of variability in relation to the mean of the population. In our model each node shall represent each petrol station with specific coefficient of variation CV_i ($i = 1, \dots, n$). Final demands at the petrol stations are assumed to be random variables (stochastic in nature) and no assumption is made about the inventory policy at an individual station. The inventory position of each station shall be analyzed and joint

inventory and routing decisions shall be made to avoid any stock-outs and minimize the total expected cost of transportation in each scenario (which is considered as linear function of total travelling distances). The inventory cost of LPG at stations shall be neglected as the cost of LPG is rather low (comparing e.g. to other liquid fuels) and usually stations' vessels capacities are relatively low.

5. NUMERICAL EXPERIMENTS

For the computational experiments real data has been taken into consideration. In our approach we first performed VRP computations applying optimization model for the 1st day, then we performed stochastic demand simulation for the 1st day, then we performed VRP computations for the 2nd day again and stochastic demand simulation for the 2nd day and again the same sequence for the 3rd day (see **Figure 2**).

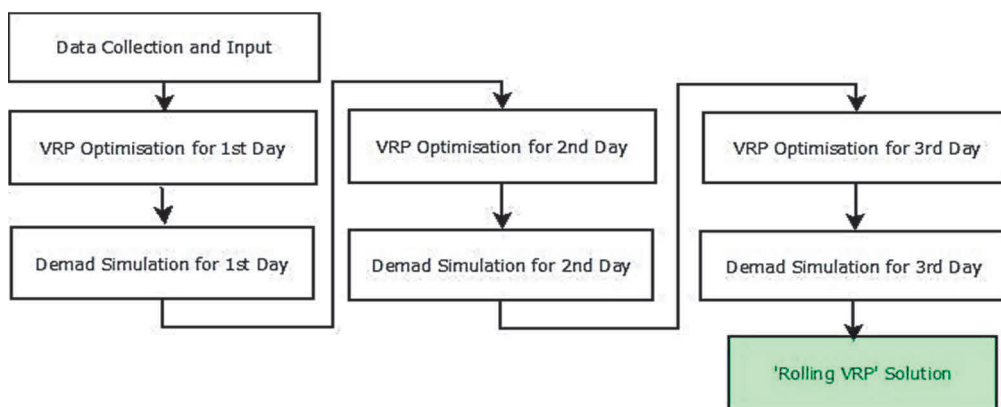


Figure 2 Overview schema of 'Rolling VRP' simulation-optimization integrated approach

In the second step we performed IRP computations applying optimization model for the first 3 days, then we performed stochastic demand simulation for the 1st day, then we performed IRP computations for the days 2-3-4 and stochastic demand simulation for the 2nd day and again we performed IRP computations for the days 3-4-5 and stochastic demand simulation for the 3rd day (see **Figure 3**).

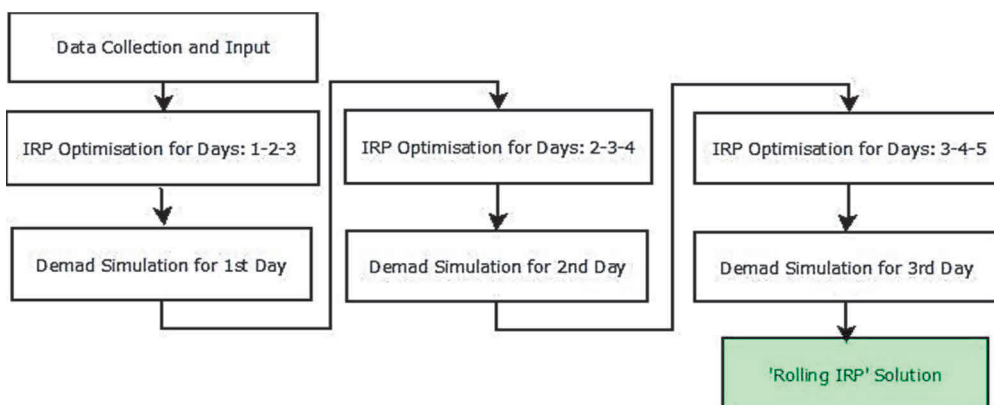


Figure 3 Overview schema of 'Rolling IRP' simulation-optimization integrated approach

Our computations were performed on real-life data of LPG distribution within one region in Poland for a given gas supplier. The region consists of one depot and 36 gas stations 24/7 open. Traditionally the customer uses from 2 to 3 truck to organize deliveries to the stations during 7 days per week and considers cost efficiency (number of kilometers per 1000 L delivered) as key point.

All computations were performed according to the following data:

- number of customers n: 36;

- number of depots: 1;
- planning horizon: 3 periods;
- demand distributions: mean demand is generated as an integer random number following a discrete triangular distribution with lower limit 10% and upper limit 85%;
- product availability at the depot: always;
- maximum inventory level: 85% due to LPG distribution specific limitations;
- starting inventory level: randomly generated;
- vehicle capacity: 36000 l.

The key performance indicators (KPI's) of our model are:

- number of kilometers per 1000 L of LPG delivered - α [km / 1000 l]
- number of stock-outs occurred at any station within the distribution period - ψ .

Our aim is to minimize number of kilometers per 1000 L of LPG delivered and avoid any stock-out at any station. Simultaneously we control the aggregated volume of LPG in whole distribution network of stations: we compare the total daily sales of all stations with the volume of gas delivered to the stations within one day. Our aim is to avoid situation when the aggregated volume of LPG in whole distribution network would drop below a certain critical level which shall oblige to use much higher number of trucks and drivers to prevent the supply chain from general inefficiency. For our computations we first consider small number of trucks (2) to perform the deliveries to all stations. However shall the number of trucks used be insufficient (an integer solution to optimize not existing), we increase number of trucks (e.g. to 3) to run our optimization. For each of data sets we have generated simultaneous computational experiments of joint usage of proposed optimization and simulation models. We first provide results for combined use of simulation and VRP optimization models for planning horizon of three days. For different coefficient of variation levels we present number of stock-outs, solution of costs / travelled distance and number of vehicles used to perform deliveries.

Table 1 Results of optimization / simulation experiments for planning horizon $k=1$ for three days (VRP variant)

CV [%]	Day			Sum	Day			Sum	Day			Sum	Day			α [km / 1000 L]
	1	2	3		1	2	3		1	2	3		1	2	3	
	stock-outs ψ				Distance travelled [km]				Liters of LPG delivered				No. of routes / vehicles			
5	0	0	0	0	326	1025	1520	2871	34997	68400	100807	204204	1	2	3	14.1
10	0	0	0	0	326	1025	1134	2485	34997	68400	70556	173953	1	2	2	14.3
15	0	0	0	0	326	1025	1040	2391	34997	72000	68400	175397	1	2	2	13.6
20	0	0	0	0	326	1025	1415	2766	34997	72000	68400	175397	1	2	2	15.8
25	0	0	0	0	326	1025	1552	2903	34997	70997	72000	177994	1	2	2	16.3
30	0	0	0	0	326	1025	1157	2508	34997	72000	70556	177553	1	2	2	14.1
35	1	0	1	2	326	999	1564	2889	34997	68400	72000	175397	1	2	2	16.5
40	1	1	1	3	326	997	1071	2394	34997	70556	72000	177553	1	2	2	13.5
45	1	1	1	3	326	968	1609	2903	34997	68400	72000	175397	1	2	2	16.6
50	2	1	1	4	326	928	1125	2379	34997	72000	68400	175397	1	2	2	13.6

Some conclusions can be drawn from **Table 1**. For coefficient of variation values below 30% we do not observe any stock-outs and number of stock-outs grows for higher values of CV.

In **Table 2** we provide results for combined use of simulation and IRP optimization models for planning horizon of three days. For different coefficient of variation levels we present number of stock-outs, solution of costs / travelled distance and number of vehicles used to perform deliveries.

Table 2 Results of optimization / simulation experiments for planning horizon $k=1$ for three days (IRP with rolling horizon variant)

CV [%]	Day			Sum	Day			Sum	Day			Sum	Day			α [km / 1000 L]
	1	2	3		1	2	3		1	2	3		1	2	3	
	stock-outs ψ				Distance travelled [km]				Liters of LPG delivered				No. of routes / vehicles			
5	0	0	0	0	793	1087	1179	3059	68400	68400	72000	208800	2	2	2	14.7
10	0	0	0	0	793	1095	1177	3065	68400	68400	70556	207356	2	2	2	14.8
15	0	0	0	0	793	1332	1451	3576	68400	72000	97200	237600	2	2	3	15.1
20	0	0	0	0	793	1095	1410	3298	68400	72000	108000	248400	2	2	3	13.3
25	0	0	0	0	793	1432	1089	3314	68400	104400	72000	244800	2	3	2	13.5
30	0	0	0	0	793	1080	1490	3363	68400	72000	104400	244800	2	2	3	13.7
35	0	0	0	0	793	1623	1060	3476	68400	107434	72000	247834	2	3	2	14.0
40	0	0	0	0	793	1452	1163	3408	68400	104400	72000	244800	2	3	2	13.9
45	0	0	0	0	793	1456	1044	3293	68400	108000	72000	248400	2	3	2	13.3
50	0	1	0	1	793	914	1542	3249	68400	72000	100800	241200	2	2	3	13.5

Some conclusions can be drawn from **Table 2**. For coefficient of variation values below 45% we do not observe any stock-outs and number of stock-outs grows for higher values of CV.

6. CONCLUSIONS AND FURTHER RESEARCH DIRECTIONS

We can now evaluate the impact on number of stock-outs and the final solution cost of usage of the two approaches (i.e. VRP method and IRP with rolling horizon method). Considering results of our computational experiments we observed important reduction of stock-outs in case of applying IRP with rolling horizon method. It was caused by additional (non-standard) condition to the model applied which allows high level of trucks utilization (i.e. 90% of truck capacity to be used). Thus deliveries in case of IRP with rolling horizon method were performed also to the stations that were not far from the distribution trip and allowed reasonable unloading in order to avoid any backhaul of LPG to the depot. This practice is common in real-life applications of LPG deliveries in Poland.

The KPI ratio number of kilometers per 1000 L of LPG delivered is the second element to be considered. In our computational experiments for 20% level of CV we observed better results for VRP with rolling horizon approach but for higher values of CV we observed more efficient distribution for IRP with rolling horizon approach. We can draw conclusion that short planning horizon (only 1 period for VRP method) provides

positive results only for low values of coefficient of variation. One of the most interesting parameters is number of trucks used for the distribution. For the VRP with rolling horizon model we needed more than 2 trucks only in 1 case however for the IRP model it occurred 8 times. In our model we analyzed impact of stochastic variations of only one parameter: demand. In real-life systems stochastic variations concern many other factors like:

- service time at the station
- travel time (due to congestion, technical failures, traffic inspections etc.)
- waiting time at depots (queues, product shortage), etc.

For further research our aim is to perform computational experiments of combined use of simulation and optimization models for simultaneous variations of different distribution parameters.

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