PRODUCTION PROCESS RELIABILITY MODELLING BASED ON THE MARKOV PROCESS IMPLEMENTATION

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Abstract

The paper discusses a problem of reliable performance of production processes. Reliability analyses of production systems regard considering many different factors and requirements. Thus, based on the developed multidimensional definition of production process reliability the seven-state Markov model of production process performance is presented. Short sensitivity analysis of the developed model is also given. The work ends up with summary and directions for further research.

Keywords: Production process, reliability modelling, Markov process

1. INTRODUCTION

The manufacturing sector faces many challenges in the era of globalization at today's highly competitive market. Complexity in taking decisions due to e.g. shorter life cycles of products, shorter delivery times, high product quality expectations, as well as immense availability of information, randomness in the system performance, or heterogeneity in occurred events all make that manufacturing system / process reliability modelling is a difficult task [1, 2].

The main target of the production system is *to operate appropriately with the maximal production rate or capacity and acceptable quality of the products* [3]. Following this, the main problems investigated in the area of production system / process reliability regard to maintenance optimization and inventory modelling issues. In this area the literature survey is quite vast and presented e.g. in [3]. There are also several studies related to reliability, availability and maintainability (RAM) of manufacturing systems (see e.g. [4, 5] for comprehensive literature review). Furthermore, another important problem in many industrial applications is the redundancy optimization aimed at system structure configuration designing taking into account cost or / and reliability criteria (see e.g. [6]). This issue may be also connected with reliability importance analysis implementation (see e.g. [7]). Moreover, the last problem in the analysed research area is the performance evaluation of production systems. The performance measurement of manufacturing systems is mostly focused on five types of performance objectives based on cost, flexibility, speed, dependability and quality [2]. The detailed literature review in this area may be found e.g. in [2, 8, 9].

Many different approaches and aspects of production process reliability which are under investigation in the analysed literature make this research area still demanding further and extensive investigation. Following this, based on the developed multidimensional production process reliability definition (see [9]), authors focus on the development of seven-stated Markov model for production process reliability assessment. The presented article gives the first attempt for development and investigation of production process reliability assessment method that helps decision managers in their every-day work.

2. PRODUCTION PROCESS RELIABILITY

Based on the literature review given in [9], authors propose the production process reliability definition as *an ability of a production system to completely fulfil the production plan of fully valuable final products in a specified period of time under stated conditions*. The given process conditions regard to the five main areas [9]:

- machines and equipment performance and their failures occurrence possibility,
- maintenance and logistic support infrastructure performance and their failures occurrence possibility,
- information flows and information reliability,
- possibility of the occurrence of unwanted random hazards / threats (internal and external type),
- processes of decisions making by policy makers and human factor reliability.

This definition is also compatible to maintenance theory, where the prime objectives are to ensure the system function, ensure system life, ensure safety, and ensure human well-being (see [10] for more information).

The complex reliability analysis should cover the investigation all of the mentioned areas providing a valuable contribution to processes performance improvement. To satisfy these requirements, the multi-state reliability theory should be implemented, where the process may assume many states ranging from perfect functioning to complete failure. Following [6] the stochastic process approach, based on Markov processes, may be used.

Let's consider a manufacturing system under continuous monitoring. It's production process may be in one of seven defined reliability states, depending on the production plan and process conditions performance (two states marked at grey are not possible to occurr in the real-life systems and are excluded from the reliability analysis) (**Table 1**). In the **Table 1** the production process state is given as a pair: the first give the possibility of production plan performance, while the second one the process condition state.

Table 1 Production process states general characteristic

State no.	Plan performance	Process condition performance	Production process state
0	Perfect	Perfect	UP state
1	Perfect	Partially imperfect	UP state
2	Perfect	Imperfect	DOWN state
3	Partially imperfect	Perfect	DOWN state
4	Partially imperfect	Partially imperfect	DOWN state
5	Partially imperfect	Imperfect	DOWN state
6	Imperfect	Perfect	DOWN state
7	Imperfect	Partially imperfect	DOWN state
8	Imperfect	Imperfect	DOWN state

The perfect performance of the production plan means that there are manufactured fully valuable products in right quantity and without any delays. Partially imerfect plan performance may regard to some time delays or problems with right quality and quantity. Imperfect performance of production plan means that based on the available resources there is no possibility to fullfil the defined plan in the defined period of time. The production process conditions performance perfectly if there is no occurred problems in machines, human factor, and support asset performance, the information is perfect (available and reliable), and there is no additional unwanted hazard events identified. Partially perfect performance of production process conditions regards usually at least to have reliable production machines and available human and information resources to fulfill the production plan. These three resources are usually critical for any manufacturing system flexible and reliable performance.

Let's also assume that production process experiences random failures in time and each failure entails a random duration of repair before the manufacturing system is put back into service. System after repair is as-good-as-new. Let's also assume that any information about failures in the system is reliable and comes immediately.

3. SEVEN-STATE MARKOV MODEL

For the analysis of the production process performance, we can associate its performance to a Markov seven-state model, on the basis of the defined set of states (**Figure 1**).

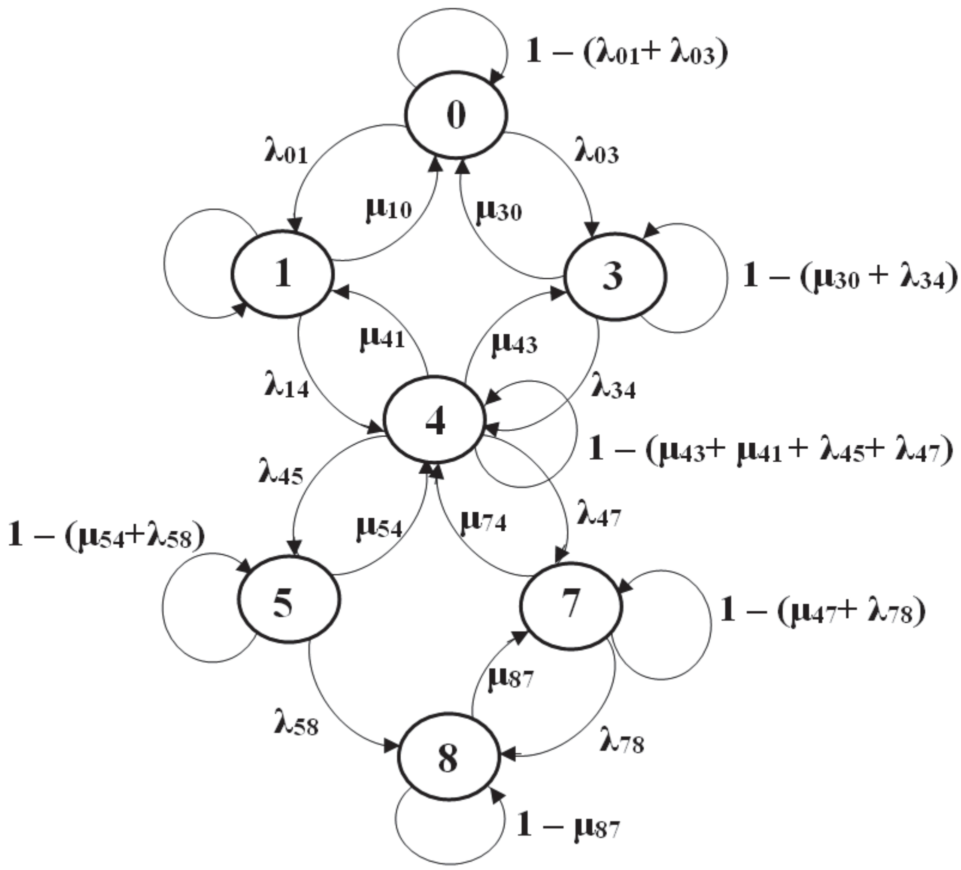

Figure 1 Markov seven-state model

The graph describes behaviour of the production process with two successful (0, 1) and five mutually exclusive failure states (3, 4, 5, 7, 8) as shown in **Figure 1** and **Table 1**. The modelled process is repaired from the states: 3, 4. Moreover, to make the model more understandable there is presented its detailed interpretation.

Let's make an assumption that the production process at the beginning is operable. This means, that it is in UP state and can perform production plan perfectly in any time when called. This also means, that there are all necessary production process conditions satisfied. In the next moment, one of the necessary resources may fail (e.g. the supply delivery will be delayed but there are available spare raw materials), but there still may be possibility to perfectly fulfil the production process (transition from the state 0 to the state 1).The state 1 is defined as the UP state of the production process, taking into account the its presented reliability definition. In the next moment, process may pass from the state 1 to the state 0 - when the necessary process conditions recovery is finished. On the other hand, process, being in the state 0, may pass to the state 3, which means that the production plan is performed partially imperfect despite satisfying all the process conditions. This state is the DOWN state of the production process.

In the next time moment, production process may pass from the states 1 or 3 to the state 4 - when both, production plan and production conditions are partially imperfect. The state 4 is the DOWN state of production process. Later, when the process conditions are imperfect (e.g. lack of human resources or machines and support infrastructure failure), process may pass from the state 4 to the state 5. The transition from the state 4 to the state 7 means that the partially operable process conditions are not enough to perform production plan, so there is no possibility to produce the right quantity of fully valuable products in the defined period of time.

In the situation, when process conditions and production plan are imperfect, at first the necessary process resources has to be restored. This is connected with the necessity of necessary maintenance assets supply to failed production system. This situation is reflected by the transition from the state 8 to the state 7, and the lack of transition from the state 8 to the state 5.

Following [11] we can formulate the probability expression for the model. Thus:

$$\begin{cases} P_0'(t) = -(\lambda_{01} + \lambda_{03})P_0(t) + \mu_{10}P_1(t) + \mu_{30}P_3(t) \\ P_1'(t) = \lambda_{01}P_0(t) - (\mu_{10} + \lambda_{14})P_1(t) + \mu_{41}P_4(t) \\ P_3'(t) = \lambda_{03}P_0(t) + (\mu_{30} + \lambda_{34})P_3(t) + \mu_{43}P_4(t) \\ P_4'(t) = \lambda_{14}P_1(t) + \lambda_{34}P_3(t) - (\mu_{43} + \mu_{41} + \lambda_{45} + \lambda_{47})P_4(t) + \mu_{54}P_5(t) + \mu_{74}P_7(t) \\ P_5'(t) = \lambda_{45}P_4(t) - (\mu_{54} + \lambda_{58})P_5(t) \\ P_7'(t) = \lambda_{47}P_4(t) - (\mu_{74} + \lambda_{78})P_7(t) + \mu_{87}P_8(t) \\ P_8'(t) = \lambda_{58}P_5(t) + \lambda_{78}P_7(t) - \mu_{87}P_8(t) \end{cases} \tag{1}$$

Where the following notations are used:

λ_{ij} - failure rate associated with states *i* and *j*

μ_{ji} - repair rate associated with states *j* and *i*

$P_i(t)$ - probability of being in state *i* at time *t*

$P'_i(t)$ - first derivative of $P_i(t)$ with respect to *t*

Taking into account steady-state solutions, there is possible to estimate the steady-state availability ratio K:

$$K = \lim_{t\to\infty} K(t) = \lim_{t\to\infty}[P_0(t) + P_1(t)] \tag{2}$$

4. SENSITIVITY ANALYSIS

Sensitivity analysis of the developed model was made using Markov Graph module of GRIF 2011 Software [12]. For the purposes of preliminary analysis performance, it was assumed that:

- Mean Time Between Failures for all states is the same and *MTBF* = 1 [time unit],
- Mean Time To Repair for all the states is the same and it changes: *MTTF* = 1; 10; 100 [time unit].

Thus, the parameter $\rho = \mu / \lambda$ obtains the following values: $\rho = 1$, $\rho = 0.1$, $\rho = 0.01$.

The performed analysis mainly focuses on the calculation of availability function *K*(*t*) of the system (**Figure 2**) and probability of being in the state 4 - $P_4(t)$ (**Figure 3**) - please note the different scales of time and values for each version a, b, c. The presented analysis results are obtained when all λ_{ij} parameters and all μ_{ji} parameters are the same.

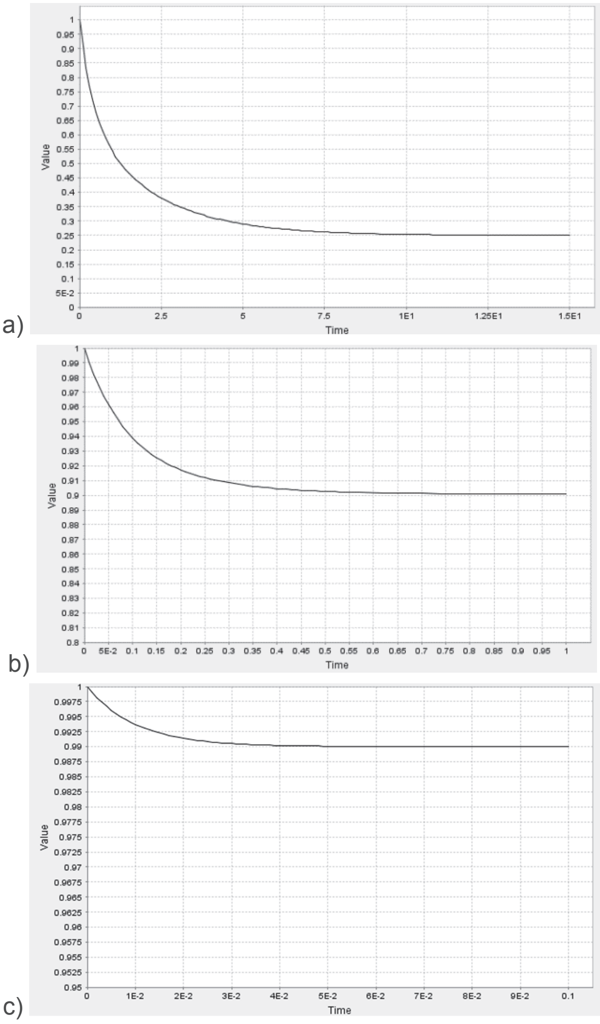

Figure 2 Availability function $K(t)$ of the system:
a) $\rho = 1$, b) $\rho = 0.1$, c) $\rho = 0.01$

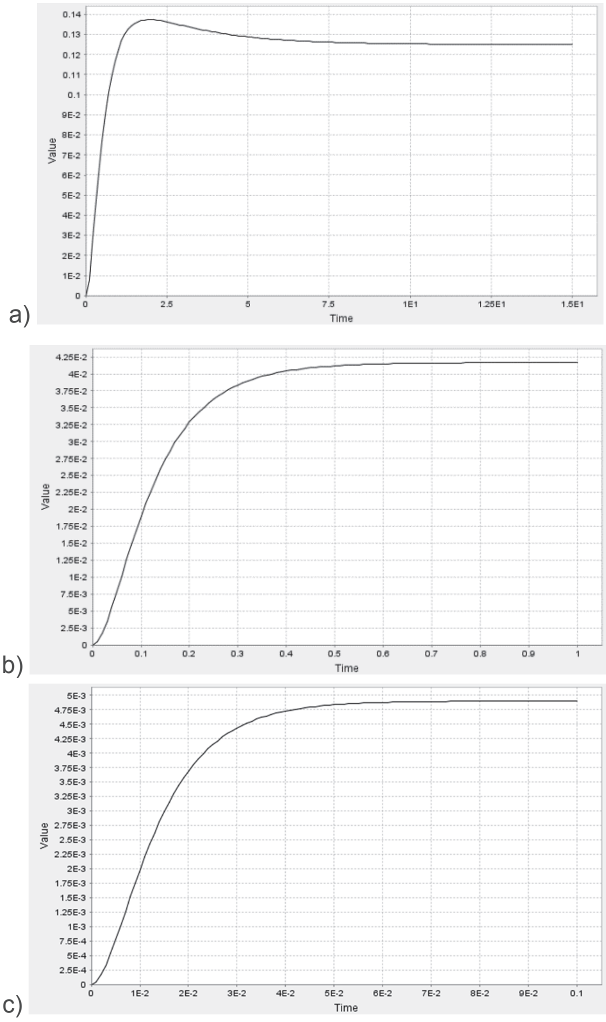

Figure 3 Probability of being in state 4 $P_4(t)$:
a) $\rho = 1$, b) $\rho = 0.1$, c) $\rho = 0.01$

The influence of mean production process down time on the availability ratio is characterized by the level of repair rates μ_{ji}. The longer the process plan and conditions are inoperable, the greater is the probability, that the production system will not be ready to perform its main functions, in its specified environment, when called for at a random point in time. As a result, the availability ratio decreases from the value of 1 to $K = 0.25$ for $\rho = 1$, by $K = 0.91$ for $\rho = 0.1$, to $K = 0.99$ for $\rho = 0.01$.

The second interesting issue is the possibility of being in the state 4 (partially imperfect plan and conditions performance). Here we also may see the same dependence - the Steady-state probability of being in the state 4 decreasing from P_4= 0.125 for $\rho = 1$, by P_4= 0.043 for $\rho = 0.1$, to P_4= 0.0047 for $\rho = 0.01$.

Also time to reach steady state depends on ρ and is from $T_\infty = 12$ for $\rho = 1$, by $T_\infty = 0.8$ for $\rho = 0.1$, to $T_\infty = 0.05$ for $\rho = 0.01$. The following examples show only the possibilities of the model sensitivity analysis.

5. SUMMARY AND CONCLUSIONS

The paper presents the preliminary step in the definition of production process model. It focuses on the developed by the authors in work [9] multidimensional definition of production process reliability. The main reason of such paper's target identification is connected with the lack of distinct definitions of production process reliability that adhere to the real-life complex manufacturing systems performance requirements.

The presented article shows the first attempt for development and investigation of production process reliability assessment method that helps decision managers in their every-day work. Taking into account the subjectivity of expert opinions, lack of information, and the variety of approaches to reliability assessment problems, authors conclude that this research area still demands further investigation and development of new complex framework for production system reliability measurement.

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