WHAT HAVE BEEN DEVELOPED FOR LOT-SIZING AND SCHEDULING PROBLEM SINCE THE EOQ MODEL WAS INTRODUCED

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Abstract

The paper is devoted to the selected heuristics for lot-sizing and scheduling problem for identical parallel machines. The lot-sizing and scheduling problem is a well-known problem in industrial engineering and logistics management, since it is crucial for inventory control and production planning. In 1913 the Economic Order Quantity (EOQ) concept was presented and the EOQ model was developed - it was a very useful tool for production and purchase planning, however, it simplified the reality significantly. Over the last century lot-sizing and scheduling problem was examined and numerous more advanced models, which consider multiple aspects of manufacturing process, were developed. In this paper a review of approaches to lot-sizing and scheduling problem are reviewed from the perspective of specific goals adopted, models developed for the problem, and methods used for solving them.

Keywords: Lot-sizing, scheduling, parallel machines, production planning, inventory control

1. INTRODUCTION

The Lot-sizing and Scheduling Problem (LSP) is a well-known problem in industrial engineering and logistics management, since it is crucial for inventory control and production planning. Since the LSP had been introduced to the scientific literature [1] numerous advanced models were developed; these formulations consider multiple aspects of manufacturing process, inventory control, and industrial logistics [2-6]. The main differences between various formulations of the lot-sizing problem are caused by aspects included (e.g. identical and non-identical parallel machines, the number of types of manufactured product, the level of safety stock, time limitations, constant, deterministic or stochastic demand) as well as the method used for solving the problem (e.g. mixed-integer linear programming (MILP), non-linear programming, heuristics, meta-heuristics) [7-20].

In this paper approaches to lot-sizing and scheduling problem are reviewed from the perspective of specific goals adopted, models developed for the problem, and methods used for solving them. Our study begins with the classical Economic Order Quantity (EOQ) concept - known also as Economic Lot Size (ELS) - introduced by F.W. Harris [1] and analyzed in details by R.H. Wilson [2]. Next, we focus on the newest advances to the parallel machines scheduling problem - due to the manifold of aspects taken into account in different approached we present a general perspective of this issue. We end our review with stochastic models that introduce uncertainty which is to be observed in the manufacturing processes. In this brief review of selected works we focus on the objective and the scientific meaning of the whole concept instead of the well-known mathematical model and formula to compute the optimal lot size. When analyzing scientific papers referred to above mentioned issues, it can be observe that there is a need for methods and tools for effective lot-sizing and scheduling in manufacturing and industrial management. Therefore, a retrospective review of the LSP development seems to be helpful to grasp the importance of the general idea of the problem.

2. THE ECONOMIC ORDER QUANTITY MODEL

In 1913 the Economic Order Quantity concept was presented in the classical article “How many parts to make at once” published in the scientific journal “Factory, The Magazine of Management”. We do not intend to present EOQ mathematical model here, since this material is covered in every elementary textbook on production planning and management science. We focus on the concept itself and its objectives just to remind that EOQ concept was a real milestone for industrial logistics. This is how the author, Ford W. Harris, introduced the lot-sizing and scheduling problem in his original paper: “Every manufacturer is confronted with the problem of finding the most economical quantity to manufacture in putting through an order. This is a general problem and admits of a general solution, and, however much it may be advisable to exercise judgment in a particular case, such exercise of judgment will be assisted by a knowledge of the general solution” [1]. The objective was to formulate a general concept that makes it possible to determine the size of lot for which the total manufacturing and holding costs are minimized (see **Figure 1**). Harris presented the EOQ model where the optimal lot size is computed with the formula that includes the unit manufacturing cost, set-up cost, unit carrying cost, and constant and continuous demand.

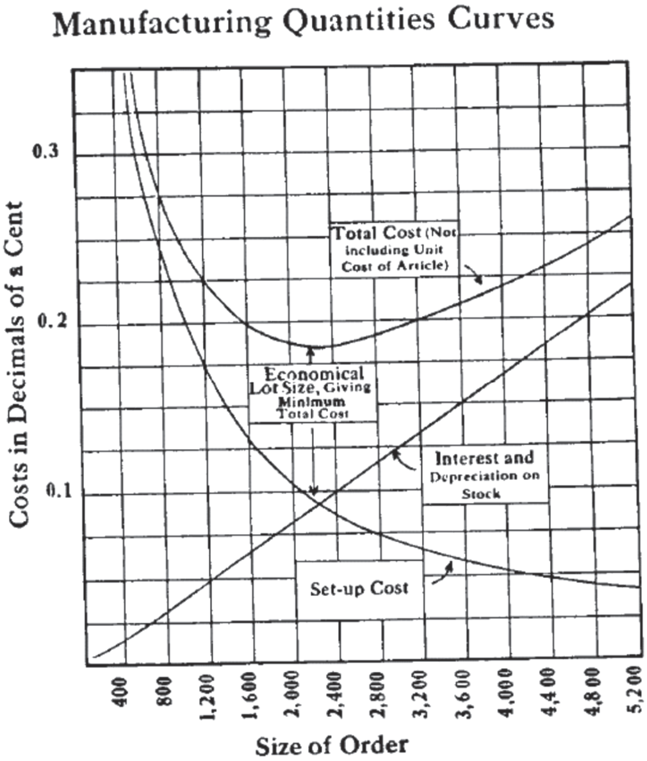

Figure 1 Manufacturing Quantities Curves - reprint of the figure illustrating the Economic Order Quantity (EOQ) concept provided by F.W. Harris in [1]

Harris was fully aware of simplifications adopted in his model, however he reached his goal - the concept is a general formulation of the problem as well as it introduced a mathematics-based tool for the lot-sizing problem, so that the optimal solution can be obtain easily and quickly. Certainly, it is possible, because a series of assumptions are adopted in this concept, what result in significant simplification of the lot-sizing and scheduling problem. The author concluded his paper as follows: “... the method given is not rigorously accurate, for many minor factors have purposely been left out of the consideration. [...] Such refinements, however, while interesting, are too fine spun to be practical. The general theory as developed hare is reasonably correct and will be found to give good results” [1]. What should be noted here is the fact that Harris developed a mathematical model that automated decision-making process on the lot size and manufacture schedule. Far reaching simplifications adopted in this model can be considered as intentionally made sacrifices, so that the solution was obtained quickly and without advanced computation machines. In result, his formulas for economic lot size, cycle length, and minimal total costs became useful tools for scheduling large-scale serial production.

3. THE DEVELOPMENT OF THE LOT-SIZING AND SCHEDULING PROBLEM

Since the paper by Harris was published the LSP has been formulated in many different ways due to different aspects of the problem; researchers have taken into account different combinations of such characteristics as: types of machines, organization of the manufacturing process (single machines or parallel machines), types of products, sequence of operations etc. Various models have been developed and different methods - both exact methods and heuristics - were employed to solve them. In this section a selection of advances to the LSP are presented. We begin with classic single machine and a single product LSP for which a dynamic programming model and a heuristic algorithm were developed. Then we briefly present considerably new approach to the single machine problem for which mixed integer linear programming models and heuristics were developed. Next, we focus on the newest advances to the parallel machines scheduling problem - due to the manifold of aspects taken into account in different approached we present a general perspective of this issue. We end our review with stochastic models that introduce uncertainty which is to be observed in the manufacturing processes.

Next milestone for LSP was an approach to the single machine single product LSP presented by H.M. Wagner and T.M. Within, where demand for the product did not has to be constant and continuous. Planning horizon is divided into planning periods, and demand is determined for each planning period. For such a formulation they developed a method based on Dynamic Programing, where the objective is to minimize total setup and holding costs. Solutions obtained with this method for LSP with a single product are optimal, however the method cannot be successfully utilized for problems where multiple products are to be scheduled [14].

Another important issue addressed by researchers is to determine production quantities over a medium size planning horizon, so that demand is met, scarce production facilities are not overloaded. A. Drexl, K. Haase and A. Kimms developed the Capacitated Lot-sizing and Scheduling Problem, which is based on the assumption that for each lot produced in a period setup cost is incurred. They emphasized that in practice the machine setup can be preserved over idle time very often, and in such cases the setup cost of a CLSP solution can be reduced by linking the production quantities of an item which is scheduled in two adjacent periods. The CLSPL is formulated by Drexl and Haase as a mixed-integer programming model, but they also developed a heuristics algorithm for the CLSPL , where a sampling method is introduced, which is backward oriented and relies on so called randomized regrets [4-6, 16].

Multiple product capacitated lot-sizing and scheduling problem was examined also by W. Kaczmarczyk, who developed MILP models for the Proportional Lot-sizing and Scheduling Problem with set-up operations overlapping multiple periods and for the Proportional Lot-sizing and Scheduling Problem with identical parallel machines. The effectivity and practical applicability of those models are assured by tightening models with constraints describing specific character of the industry to which models are dedicated. The planning horizon is divided into planning periods, and each machine capacity in each period is limited and must not be exceeded. The objective is the same as in the previously presented models: to minimize the total setup and holding costs in all periods [7, 12].

An important aspect of lot-sizing and scheduling problem is the case with parallel machines. The aim of this problem is to sequence a set of *n* jobs on *m* parallel machines in order to minimize the performance indicator. The parallel machine scheduling problem can be divided into following categories with respect to the type of machines. Machines may be identical, uniform, or completely unrelated and have different speeds; identical machines - the processing times are the same for every machine, uniform machines - machines have different speeds but each machine works at a consistent rate, unrelated machines - machines can work at different rates and a given machine can process. For parallel machines scheduling problem numerous models and heuristics were developed, e.g. genetic algorithms, artificial immune system algorithm, or vibration damping optimization algorithm [21, 22].

Table 1 Advances in lot-sizing and scheduling problem - a selective literature review

Author(s)	Paper	Year	Problem	Objectives	Method
F.W. Harris	How Many Parts to Make at Once [1]	1913	single machine, single product, single planning period,	to minimize total manufacturing and holding costs	mathematical programming
H.M. Wagner, T.M. Within	Dynamic version of the economic lot size model [14]	1958	single machine, single product, multiple planning periods,	to minimize total setup and holding costs	dynamic programming
A. Drexl, K. Haase	Proportional lotsizing and scheduling [6]	1995	single machine	to minimize total setup and holding costs	mixed-integer programming
W. Kaczmarczyk	Wybrane modele planowania wielkości i szeregowania partii produkcyjnych [7]	2011	single machine, identical parallel machines, multiple planning periods, multiple products,	to minimize total setup and holding costs	mixed-integer linear programming
E. Mehdizadeh, R. Tavakkoli-Moghaddam, M. Yazdani	A vibration damping optimization algorithm for a parallel machines scheduling problem with sequence-independent family setup times [21]	2015	parallel machines, sequence-independent family setup times	to minimize the total weighted completion time	vibration damping optimization algorithm
K. Schemeleva, X. Delorme, A. Dolgui	A memetic algorithm for a stochastic lot-sizing and sequencing problem [23]	2015	assembly line, sequencing problem, uncertainty on defective items due to the machines' imperfection, uncertainty on repair time	to maximize the probability of overall demand satisfying	memetic algorithm
S. Özpeynirci, B. Gökgür, B. Hnich	Parallel machine scheduling with tool loading [17]	2016	parallel machines, tool magazines capacity constraints, tool assignment constraints	to minimize the makespan (Cmax)	mixed integer programming, tabu search algorithms
M. Afzalirad, J. Rezaeian	Resource-constrained unrelated parallel machine scheduling problem with sequence dependent setup times, precedence constraints and machine eligibility restrictions [22]	2016	parallel machines, resource constrains, sequence-dependent setup times, different release dates, machine eligibility constraints, precedence constraints	to minimize the makespan (Cmax)	genetic algorithm, artificial immune system algotithm

Another important contribution to the development of the lot-sizing and scheduling problem is so-called lot-sizing and sequencing problem under uncertainties, which introduces stochastic processes and probability into models. The objective here is to maximize the probability of overall demand satisfying by a system in which rejects and breakdowns occur. It should be emphasized here that, when a company faces lot-sizing and scheduling problem they realize that the problem is composed of: a manufacturing line that process items of several types, an automatic storage device that stocks processed items, an assembly line assembling final products with stored items. At each stage of those processes some uncertainty may appear - mostly connected to rejects and breakdowns. Lot-sizing and scheduling problem becomes even more complicated in such circumstances and models dedicated to it are NP-hard. Well-known approach to this problem is a 3-level decomposition of the lot-sizing and scheduling problem with abovementioned uncertainties, where the 1st level is a complete enumeration of n possible solutions, the 2nd level is an equivalent to the Asymmetric Travelling Salesman Problem, and the 3rd is an extension of the Knapsack problem. It should be emphasized here that the 2nd and 3rd levels are NP-hard, therefore even if MILP model is developed for the lot-sizing and sequencing problem under uncertainties, it was necessary to employ other methods to solve this problem. Dolgui et al. developed a method using Dynamic Programming procedure, and Schmeleva et al. introduced a special genetic algorithm called memetic algorithm [23, 24].

4. CONCLUSIVE REMARKS

In this paper the brief review of selected directions of advances in lot-sizing and scheduling problem is presented. Over 100 years manufacturing process has changed a lot, as well as scientific approach to manufacturing optimization. From the general concept of computing the optimal lot size the lot-sizing and scheduling problem has evolved in various formulations and advanced models.

It should be noted that when more advanced models for lot-sizing and scheduling problem are being developed formulations known from assignment problem or travelling salesman problem appear. It results in NP-hardiness of problems. For that reason for many years heuristic algorithms were developed, since they can find solutions with acceptable optimization gaps in reasonably short time. It matters, because lot-sizing and scheduling model is the core of computer-aided tools for generating manufacturing schedules. In many cases parameters change unexpectedly and it is needed to re-schedule production plan quickly. However, nowadays it is to be observed utilization of exact methods, since the increase of computing power makes it possible to get results with such methods in reasonably short time.

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REFERENCES

[1] HARRIS F.W. How many parts to make at once. Factory. The magazine of management, Vol. 10, No. 152, 1913, pp. 135-136.

[2] WILSON R.H. A Scientific Routine for Stock Control. Harvard Business Review, Vol. 13, No. 1, 1934, pp. 116-128.

[3] KARIMI B., FATEMI GHOMI S., WILSON J. The capacitated lot sizing problem: a review of models and algorithms. Omega, Vol 31, pp. 365-378.

[4] HAASE K., KIMMS A. Lot sizing and scheduling with sequence-dependent setup costs and times and efficient rescheduling opportunities. International Journal of Production Economics, Vol. 66, No. 2, 2000, pp. 159-169.

[5] DREXL A. KIMMS A. Lot sizing and scheduling - Survey and extensions. European Journal of Operational Research, Vol. 99, No. 2, 1997, pp. 221-235.

[6] DREXL A. HAASE K. Proportional lotsizing and scheduling. International Journal of Production Economics, Vol. 40, No. 1, 1995, pp. 73-87.

[7] KACZMARCZYK W. Wybrane modele planowania wielkości i szeregowania partii produkcyjnych. Wydawnictwa AGH: Kraków, 2011.

[8] KSIĄŻEK R., GDOWSKA K. Algorytmy heurystyczne wyznaczania wielkości produkcji dla znanego harmonogramu przezbrojeń dla zadania planowania wielkości i szeregowania partii produkcyjnych. Logistyka, Vol. 4, 2014, pp. 4597-4608.

[9] GDOWSKA K., KSIĄŻEK R. The structure of the genetic algorithm of lot-sizing and scheduling problem formulated as capacitated lot sizing problem. Logistyka , Vol. 2, 2012, pp. 559-565.

[10] DELLAERT N., JEUNET, J. Solving large unconstrained multilevel lot-sizing problems using a hybrid genetic algorithm. International Journal of Production Research, Vol. 38, No. 5, 2000, pp. 1083-1099.

[11] BERALDI P., GHIANI G., GRIECO A. GUERRIERO E. Rolling-horizon and fix-and-relax heuristics for the parallel machine lot-sizing and scheduling problem with sequence-dependent set-up costs. Computers & Operations Research, Vol. 35, No. 11, 2008, pp. 3644-3656.

[12] KACZMARCZYK W. Modele PLC planowania wielkości i szeregowania partii z identycznymi liniami równoległymi. Zeszyty Naukowe Politechniki Ślaskiej. Automatyka, Vol. 144, 2006, pp. 23-32

[13] WOLSEY L.A. Uncapacitated Lot-Sizing Problems with Start-Up Costs. Operations Research, Vol. 37, No. 5, 1989, pp. 741-747.

[14] WAGNER H.M., WHITIN T.M. Dynamic version of the economic lot size model. Management Science, Vol. 5, 1958, pp. 89-96.

[15] STADTLER H. Multi-level single machine lot-sizing and scheduling with zero lead times. European Journal of Operational Research, Vol. 209, No. 3, 2011, pp. 241-252.

[16] QUADT D., KUHN H. Capacitated lot-sizing and scheduling with parallel machines, back-orders, and setup carry-over. Naval Research Logistics, Vol. 56, No. 4, 2009, pp. 366-384

[17] ÖZPEYNIRCI S., GÖKGÜR B., HNICH B. Parallel machine scheduling with tool loading. Applied Mathematical Modelling, Vol. 40, No. 9-10, 2016, pp. 5660-5671.

[18] MARINELLI F., NENNI M.E., SFORZA A. Capacitated lot sizing and scheduling with parallel machines and shared buffers. A case study in a packaging company. Annals of Operations Research, Vol. 150, No. 1, 2007, pp. 177-192.

[19] LEE C.-H., LIAO C.-J., CHAO C.-W. Unrelated parallel machine scheduling with dedicated machines and common deadline. Computers & Industrial Engineering, Vol. 74, 2014, pp. 161-168.

[20] LEE C.-H., LIAO C.-J., CHUNG T.-P. Scheduling with multi-attribute setup times on two identical parallel machines. International Journal of Production Economics, Vol. 153, 2014, pp. 130-138

[21] MEHDIZADEH E., TAVAKKOLI-MOGHADDAM R., YAZDANI M. A vibration damping optimization algorithm for a parallel machines scheduling problem with sequence-independent family setup times. Applied Mathematical Modelling, Vol. 39, No. 22, 2015, pp. 6845-6859.

[22] AFZALIRAD M., REZAEIAN J. Resource-constrained unrelated parallel machine scheduling problem with sequence dependent setup times, precedence constraints and machine eligibility restrictions. Computers & Industrial Engineering, Vol. 98, 2016, pp. 40-52.

[23] SCHEMELEVA K., DELORME X.,DOLGUI A., A memetic algorithm for a stochastic lot-sizing and sequencing problem. IFAC - Papers On Line, Vol. 48, No. 3, 2015, pp. 1809-1814.

[24] DOLGUI A., EREMEEV A.V., KOVALYOV M.Y., KUZNETSOV P.M. Multi-product lot-sizing and scheduling on unrelated parallel machines to minimize makespan. IFAC Proceedings Volumes, Vol. 42, No. 4, 2009, pp. 828-833.