FORMAL MODEL OF WAREHOUSE PROCESS WITH FLOWS ITINERARIES

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Abstract

The paper presents elements of formal model of warehouse process based on selection of technologically plausible material flow itineraries according to time of realization and space, labour and equipment resources constrains. Model includes structure of supplies and shipments from warehouse facility and types of materials. It can be applied to assessment of efficiency or reliability of warehouse processes depending on the formulated criteria function.

Keywords: Warehouse process, modelling, process itineraries

1. INTRODUCTION

Warehouses are key elements of logistics networks. Without regard to the location and type warehouses perform logistics processes buffering, moving and transforming materials. Warehouse process is a sequence of transformations effected on material flows to change incoming materials into materials dispatched to the clients. Typical warehouse processes consists of standard, repeatable logistics operations ([4], [7], [9]).

Warehouse processes are modelled to investigate design variants of warehouse facilities and to manage warehousing operations [10]. Underlying pace of developing modern warehouse management systems is to include „what if?“ analyse tool supporting decision making which needs adjusted formal models representing core feature of warehousing operations. Warehouse processes and their models are widely discussed in literature. Dominant standpoints for that are efficiency, reliability and responsiveness, especially when it comes to supply chain configuration ([1], [3]). For over 50 years this problems are discussed on different levels (ie. [2], [5], [6], [8], [11]) and different mathematic models were introduced.

The paper presents assumptions and general formulations for formal model of warehousing process with itineraries of material flow as convenient organization method. The itinerary is defined as realization of particular implementation of material flow through the warehouse facility. Determination of itinerary depends on availability of technical and human resources and availability of free space and ordered materials at a certain time.

2. THE MODEL OF WAREHOUSE PROCESS ITINERARIES

2.1. Assumptions

Representing material flows in mathematical model requires simplifications on the one hand, and must contain all substantial features of modelled flows on the other hand [9], [11]. The model is constituted by basic elements representing functional and physical structure of the warehouse, human and technical resources, material flows, organization and information system. As it was discussed in [4] model of any warehouse can be noted as ordered six:

$$\mathbf{MCD} = \langle \mathbf{S}, \mathbf{R}, \mathbf{Q}, \mathbf{P}, \mathbf{O}, \mathbf{I} \rangle \quad (1)$$

where:

S - *structure* mapping functional and physical conditions of warehouse facility, based on hierarchical division of warehouse into subsequent tiers of functional areas grouping locations (addresses),

R - *resources* representing human and technical resources (means of internal transport and equipment).

Q - *material flows* defining volumes and structure of material flows and range of transformations related to shipment orders and supplies,

P - *logistics process* mapping sequence of material flow transformations,

I - *information system* representing mechanisms triggering, controlling and confirming material flows,

O - *organization* defining ways of transforming material flows in current situation.

The model of warehouse processes bases on foregoing model of warehouse facility:

1) Warehouse is divided into functional areas performing consecutive phases of warehouse process.
2) Warehouse process is split into phases of *supplies* (purchased materials are moved from loading docks to storage areas) and *shipments* (ordered materials are moved from storage areas to loading docks to be shipped). Material flows in both phases are non-correlated in a short time (like day) but are performed by shared resources in the same functional areas.
3) The basic flow unit is *purchase-line* for supplies and *order-line* for shipments. Line is a part of a sales order or purchase order that specifies the detailed information about requested (single) item.
4) The line can be realized in different ways upon availability of equipment, ordered materials and space. Plausible variants of line realization are called *itineraries*.
5) Each itinerary has defined probability of occurrence, wherein some itineraries are more plausible (primary processes) then other (auxiliary and emergency processes).
6) Probability of itinerary selection can be increased by equipment modernization, re-organization and improving storage capacity.
7) Itinerary is composed of sections of determined duration. Duration time can be set as random variable dependent on availability of resources, materials and space in current system state.
8) Aggregated duration time of itinerary is used to set efficiency and reliability parameters of warehouse.

2.2. Functional areas and material flow transformations

Warehouse functional areas correspond to places where parts of warehouse process are performed. Functional areas most commonly use specialized technologies of material flow handling or transforming which results from process requirements. Functional areas group physical locations (addresses) which are places where units of material are directly put or retrieved from. Locations in functional area can be divided into sub-sets of locations engaged in particular stages of warehouse process. In that way set of functional areas $\boldsymbol{FA} = \{1, 2, ..., fa, ..., FA\}$ is decomposed (**Figure 1**) into set of nodes $\boldsymbol{SF} = \{1, 2, \ldots, sf, \ldots, SF\}$.

Nodes extracted from warehouse functional areas are categorized by four sets $\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C} \cup \boldsymbol{V} = \boldsymbol{SF}$ where:

A - set of sources of material flows entering warehouse (supplies) a, $a \in \boldsymbol{A}$,

C - set of mouths of material flows exiting warehouse (shipments) c, $c \in \boldsymbol{C}$,

B - set of nodes representing storage areas (long-term storage) which are both; internal mouths and sources of material flows b, $b \in \boldsymbol{B}$,

V - set of intermediate nodes v, $v \in \boldsymbol{V}$ (**Figure 1**).

To represent material flow structure the following sets are defined:

- set of material groups $\boldsymbol{GM} = \{1, 2,\ldots, gm,\ldots, GM\}$ representing types of goods or / and package forms,
- set of time moments $\boldsymbol{T} = \{1, 2, \ldots, t, \ldots, T\}$,
- set of types of transport equipment and machines $\boldsymbol{U} = \{0, 1, 2, \ldots, u, \ldots, U\}$,
- set of labour categories $\boldsymbol{H} = \{0, 1, 2,\ldots, h, \ldots, H\}$.

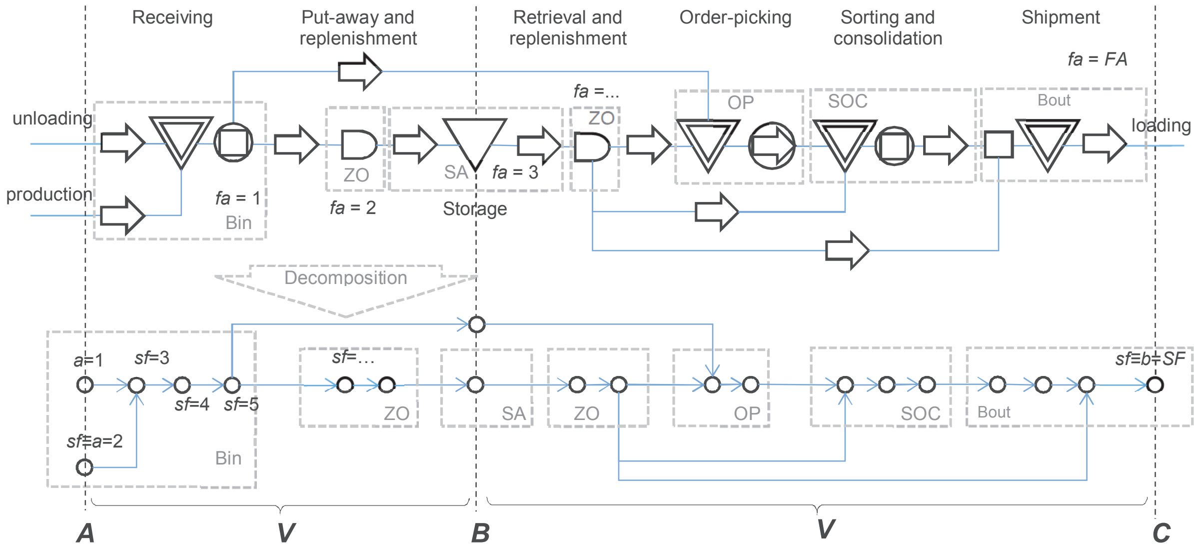

Figure 1 Basic warehouse process decomposition

Each unit of material of *gm*-th group entering the facility is subjected to sequence of transformations whose ultimate goal is to ship it to customer in accordance with order. Transformations are interpreted as transitions between pairs of structural nodes. For simplicity all transformations are divided into two groups: 1 / *movement* within functional area and between them, 2 / *processing* (conversion), including: simple production, breaking and consolidation, control and identification (delay) and buffering (awaiting and short term storage). Type of transformation is formally noted as $i(sf, sf') = 1$, if material is *moved* between nodes *sf* and *sf'*, $i(sf, sf') = 2$ if material is *converted* or *buffered* and $i(sf, sf') = 0$ if no transformation occurs between nodes (**Figure 2**). Only one connection between two nodes *sf* and *sf'* is possible. In case of larger number of transformations between nodes the deeper decomposition of warehouse structure is necessary. Buffering capacity of functional area is represented by maximal number of transformations $i(sf, sf') = 1$ to be performed at the time between nodes representing that area.

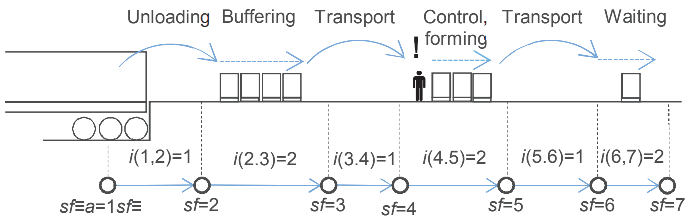

Figure 2 Transformations in warehouse process

Transformations can be divided into two groups according to the phase of warehouse process. The first group of transformations $\boldsymbol{I}_{in}$ *fills* storage areas (supplies) while second group $\boldsymbol{I}_{out}$ *empties* them (shipments). Accordingly, set of all transformations concurring to warehouse process is described as $\boldsymbol{I} = \boldsymbol{I}_{in} \cup \boldsymbol{I}_{out} = \{(sf, sf'): i(sf, sf') \in \{1, 2\}, sf \neq sf', sf, sf' \in \boldsymbol{SF}\}$.

Transformations of material flows $i(sf, sf')$, $i(sf, sf') \in \boldsymbol{I}$ are interpreted as transitions between nodes sf, $sf \in \boldsymbol{SF}$. Nodes arise as a result of decomposition of functional areas fa, $fa \in \boldsymbol{FA}$. Thus, it was assumed that the structure of logistics process in warehouse can be represented by graph $\boldsymbol{G}$: $\boldsymbol{G} = \langle \boldsymbol{SF}, \boldsymbol{I} \rangle$ where $\boldsymbol{SF}$ is a set of nodes and $\boldsymbol{I}$ is a set of edges.

Daily volumes of successive transformations depends on supplies and shipments structure. Supplies and shipments are given as random variable of known distribution {$Q_{in}(gm,t)$, $p(Q_{in}(gm,t))$} and {$Q_{out}(gm,t)$, $p(Q_{out}(gm,t))$}. Distributions are determined on the basis of historical data. It was assumed that supplies and shipments from the warehouse are independent in a short time (usually day). In the long term the structure of supplies and shipments are correlated due to the planning mechanisms.

As previously described material flow volumes are expressed by number of purchase-lines *pl*, $pl \in \boldsymbol{PL}$ (supplies) and order-lines *ol*, $ol \in \boldsymbol{OL}$ (shipments) submitted to the system. Realization of purchase and order lines may be done in different ways - depending on the availability of resources and the structure of supplies and shipments. Possible variants of line realization are noted as itineraries.

2.3. Warehouse process - itineraries

Ordered set of process itineraries corresponding to possible passes of materials through the warehouse constitute a warehouse process. Itineraries are interpreted as acceptable paths in graph ***G***. The path depends on unoccupied resources that can be assigned to the realization of each line in consecutive instants. Path in graph ***G*** is established separately for each purchase and order-line. Process itinerary is an element of logistics process determining sequence of handling operations, conversions and delays caused by buffering performed on materials specified by purchase or order line. Therefore, itinerary defines the technologies and resources applied to its realization ([9], [11]).

Each itinerary is described by sequence of nodes of which the first is the start of relation and the last one is the end of relation. Passing relation of supplied materials is defined as ordered pair of nodes (*a*, *b*) where $a \in \boldsymbol{A}$, $b \in \boldsymbol{B}$ and passing relation of shipped materials is defined as ordered pair of nodes (*b*, *c*) where $b \in \boldsymbol{B}$ and $c \in \boldsymbol{C}$. Filling and emptying processes described above can differ for different groups of materials, so any two nodes *a* and *b* or *b* and *c* can be connected by many process itineraries. The single itinerary connecting source *a* with storage area *b* is numbered as e_{in} and noted as follows: $pp_{in}(a,b,e_{in}) = \langle (a \equiv sf_1, sf_2),...,(sf_n, sf_{n+1}),...,(sf_N, sf_{N+1} \equiv b) \rangle$. Set $\boldsymbol{E}_{in}(a,b)$ gathers process itineraries for relation (*a*, *b*). By analogy single itinerary connecting storage area *b* with exit *c* is numbers as e_{out} and noted as $pp_{out}(b,c,e_{out}) = \langle (b, sf),...,(sf', sf''),...,(sf^M, c) \rangle$ while $\boldsymbol{E}_{out}(b,c)$ is a set of itineraries for relation (*b*, *c*).

Daily volumes of supplies and shipments dictate content and number of purchase and order-lines. Single line can be realized in different ways, but total number of possible itineraries is known. The probability of particular itinerary selection depends on technical and organizational factors, but itineraries representing primary (desirable) process are more likely than other. Basic factors influencing itinerary selection are: free space in buffering and storage areas (for supplies), and availability of ordered materials (for shipments).

If ordered item is not available in functional area or will not be available within specified time, probability of selection of itinerary including that area is 0. Each purchase or order-line has appointed tree of decision situations embracing all possible itineraries of realization. The exemplary tree of itineraries for single shipment line is presented in **Figure 3**. Material flow processes can be disturbed by errors and mistakes, in most cases of human origin. Detection of any error can be done at specific stages of the process. Error detection triggers corrective operations, which are elements of different itineraries.

2.4. Model parameters

In addition to previously-defined sets, each type of route and each material group must be described by additional parameters (selected):

$n(u)$ - number of available vehicles / machines of *u*-th type,
$n(h)$ - number of available workers of *h*-th labour category,

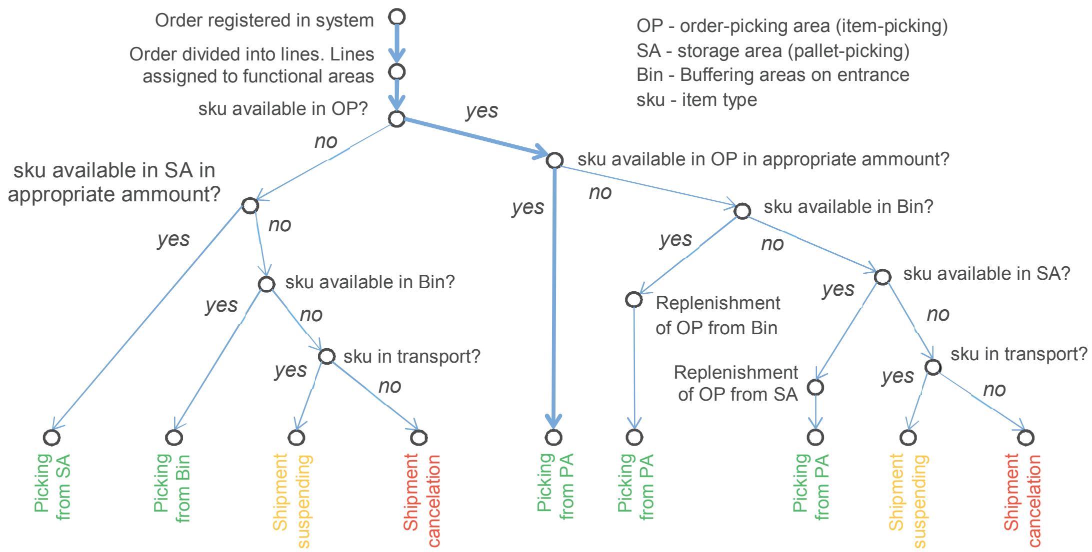

Figure 3 Tree of decision situations for selecting process itineraries - example.

$l1(u, gm, pp_{in}(a, b, e_{in})))$ - number of vehicles / machines of *u*-th type necessary to service unit of *gm*-th material under filling itinerary e_{in},

$l2(u, gm, pp_{out}(b, c, e_{out}))$ - number of vehicles / machines of *u*-th type necessary to service unit of *gm*-th material under emptying itinerary e_{out},

$l3(h, gm, pp_{in}(a, b, e_{in}))$ - number of workers of *h*-th labor category necessary to service unit of *gm*-th material under filling itinerary e_{in},

$l4(h, gm, pp_{out}(b, c, e_{out}))$ - number of workers of *h*-th labor category necessary to service unit of *gm*-th material under emptying itinerary e_{out},

$t1(gm, pp_{in}(a, b, e_{in}))$ - time of performing process itinerary e_{in},

$t2(gm, pp_{out}(b, c, e_{out}))$ - time of performing process itinerary e_{out}.

Process itinerary duration is set by times *t*((*sf*, *sf'*), *u*, *h*, *gm*, *gm'*) of particular transformations constituting that itinerary. The time depends on assigned resources, group of material and transformation type.

2.5. Decision variables

The model can use following decision variables (selected):

$x1(pl, gm, pp_{in}(a, b, e_{in}))$ - binary assignment of *pl*-th purchase-line to itinerary e_{in},

$x2(ol, gm, pp_{out}(b, c, e_{out}))$ - binary assignment of *ol*-th order-line to itinerary e_{out},

$x1(pl, gm, pp_{in}(a, b, e_{in}), t)$ - binary assignment of *pl*-th purchase-line to itinerary e_{in} in moment *t*,

$x2(ol, gm, pp_{out}(b, c, e_{out}), t)$ - binary assignment of *ol*-th order-line to itinerary e_{out} in moment *t*,

$t1(pl)$ - moment of releasing *pl*-th purchase-line to realization $t1(pl) \in \boldsymbol{T}$,

$t2(ol)$ - moment of releasing *ol*-th order-line to realization $t2(ol) \in \boldsymbol{T}$.

2.6. Technological constrains and criteria function

The model can use following constrains (selected):

1) Limited resources (available number of people and equipment in moment *t*).
2) Itinerary selection (each purchase and order-line must be assigned to itinerary).

3) Maximal itinerary queuing time (from line registration to realization begin).
4) Maximal storage capacity of functional area and availability of ordered materials.
5) Required ratio of successful itineraries to all itineraries in supply handling and shipments.
6) Maximal itinerary realization time for purchase and order-line.
7) Balanced flows between ***A*** and ***B*** and between ***B*** and ***C*** (all started itineraries must be finished).
8) Non-negative flow volumes.

The basic requirement for a process is maximal number of handled shipments and supplies:

$$\sum_{gm \in GM} \left(\begin{array}{l} \sum_{pl \in PL} \sum_{pp_{in}(a,b,e_{in}) \in E_{in}} \sum_{a \in A} \sum_{b \in B} x1\left(\mathrm{pl}, gm, pp_{in}(a,b,e_{in})\right) + \\ + \sum_{ol \in OL} \sum_{pp_{out}(b,c,e_{out}) \in E_{out}} \sum_{b \in B} \sum_{c \in C} x2\left(ol, gm, pp_{out}(b,c,e_{out})\right) \end{array} \right) \longrightarrow \max \qquad (2)$$

3. CONCLUSION

Presented elements of formal model of warehouse process can be freely developed in order to meet specific requirements of particular processes and modelling goals. Process itinerary is a basic mechanism of simulation which is efficient tool of process analyses. Proposed parameters, variables, constrains and criteria function are examples of possible application using only part of potential included in the model.

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